ACSAC 2023
December 4–8, 2023 • Austin, Texas, USA

Mostree: Malicious Secure Private Decision Tree Evaluation with Sublinear Communication

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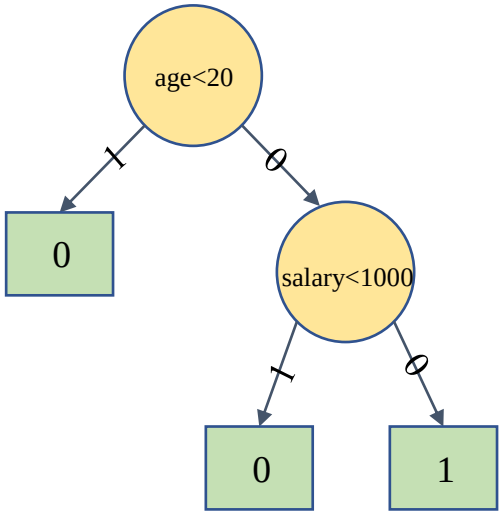
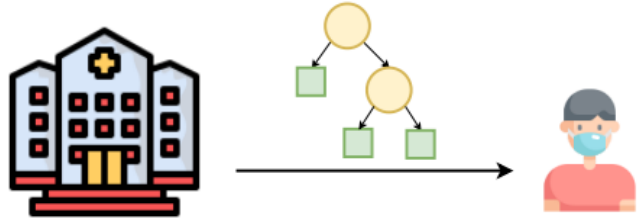
Decision Trees

- Tree structure model
  - Simple and interpretable
- Applications
  - Healthcare system
  - Spam detection
  - Credit-risk assessment
  - …



Privacy Challenges

- The decision tree model is valuable



- The feature vector is sensitive

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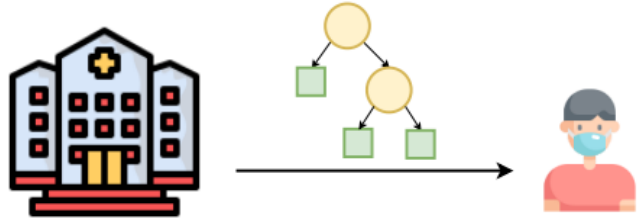


- The feature vector is sensitive

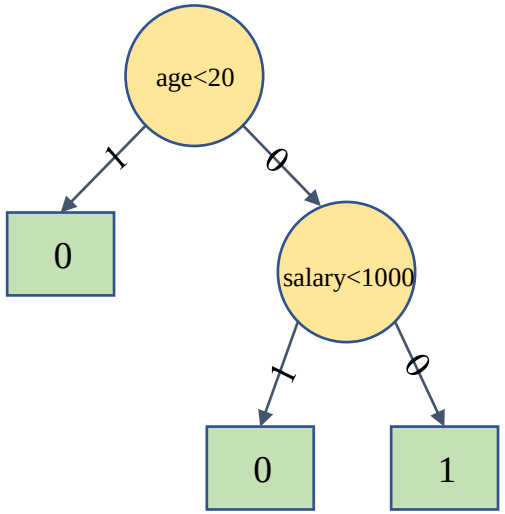
- How to perform private decision tree evaluation (PDTE)?

Sensitive Feature/Tree Information

- Feature example:

age	gender	salary
35	male	3000

- Tree example:



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- Sensitive information
  - Attribute values (e.g., age=35, gender=male,…)

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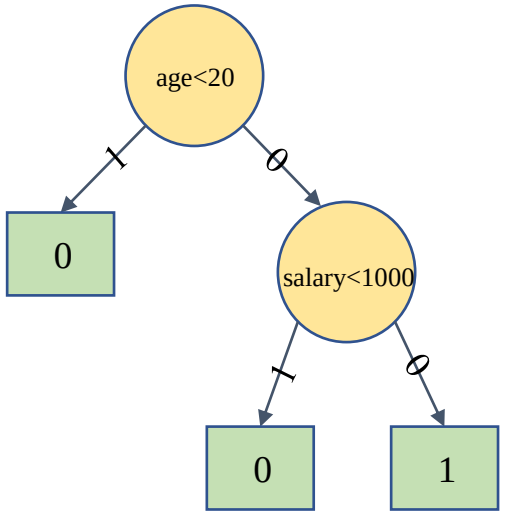

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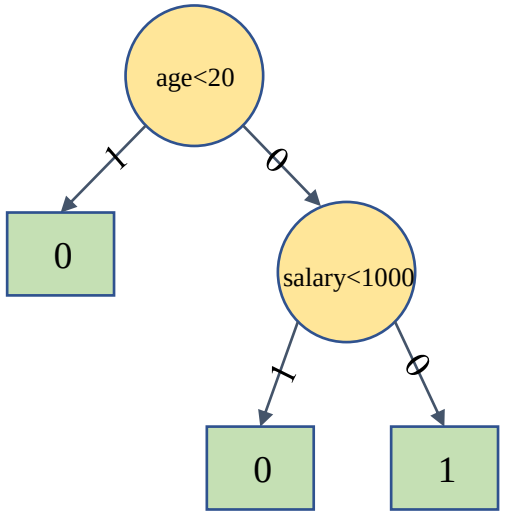

- Sensitive information
  - Attribute values (e.g., age=35, gender=male,…)
- Public information
  - Feature dimension (e.g., 3)
  - Attribute names and their organization (e.g., age→gender→salary)

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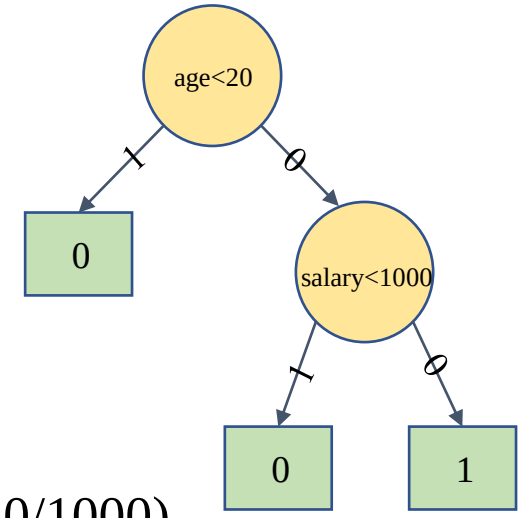

- Sensitive information
  - Threshold (e.g., 20/1000)
  - Attributes of interest (e.g., age/salary)
  - Depth of leaf nodes (e.g., 1/2)

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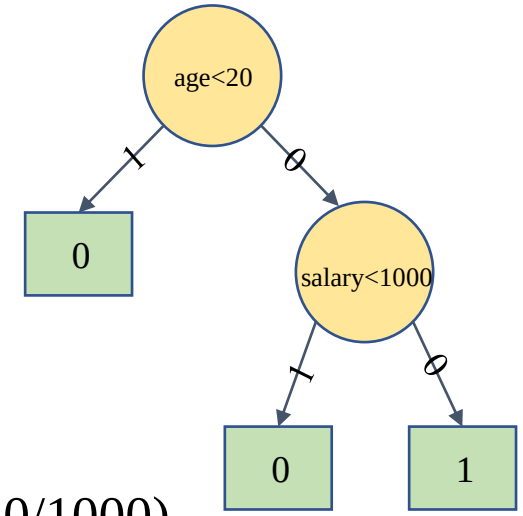

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  - Threshold (e.g., 20/1000)
  - Attributes of interest (e.g., age/salary)
  - Depth of leaf nodes (e.g., 1/2)
- Public information
  - The number of nodes (e.g., 5)
  - Maximal depth of the tree (e.g., 2)

System Model/Threat Model

CP

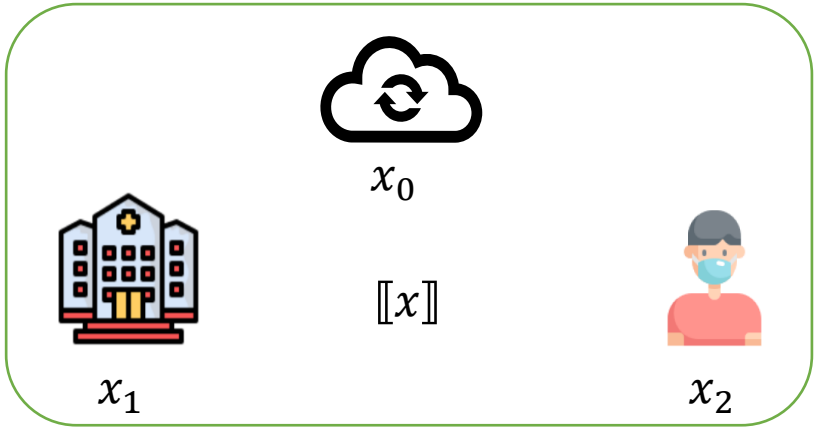
MO

FO

- System model:
  - Model owner (MO)
  - Feature owner (FO)
  - Computing party (CP)
- Threat model:
  - **Honest-majority setting**: At most one party can be **malicious**.
  - **Security goals**: Achieve *privacy* in any case, and *correctness* if the protocol completes (i.e., malicious security with abort).

Cryptographic Primitives

- $\binom{n}{n}$-secret sharing $[\![x]\!]$
  - $x = x_0 + x_1 + \cdots + x_{n-1} \bmod L$
  - $n = 2$ or 3 in this talk
  - if $n = 3$, P_0 has (x_0), P_1has (x_1), and P_2 has(x_2)



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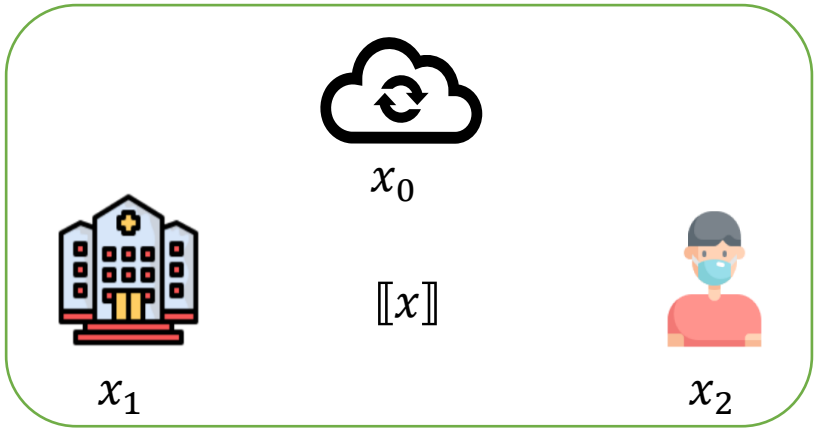
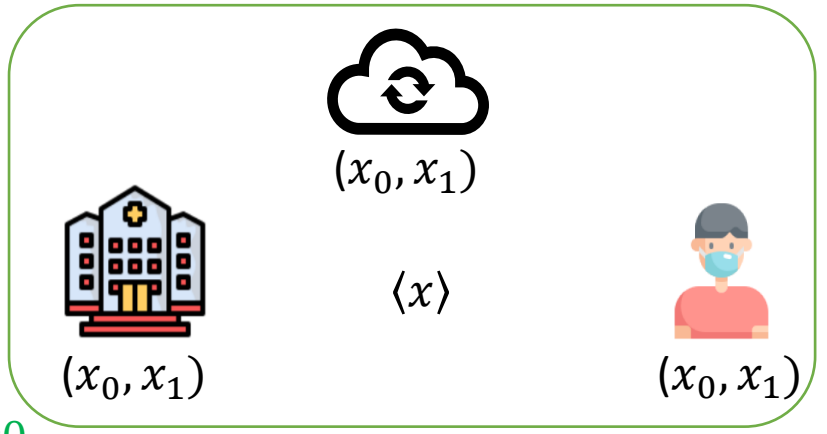
  - if $n = 3$, P_0 has (x_0), P_1has (x_1), and P_2 has(x_2)
- $\binom{3}{2}$-secret sharing $\langle x \rangle$
  - $x = x_0 + x_1 + x_2 \bmod L$
  - P_0 has (x_0, x_2), P_1has (x_1, x_0), and P_2 has(x_2, x_1)
  - Also known as replicated secret-sharing (RSS)
  - Multiplication: $\langle z \rangle \leftarrow \langle x \rangle \cdot \langle y \rangle$
    1) P_i computes $z_i \leftarrow x_i y_i + x_i y_{i-1} + x_{i-1} y_i + r_i$, where $r_0 + r_1 + r_2 = 0$.
    2) P_i sends z_i to P_{i+1} and receives z_i from P_i .
    3) P_i outputs (z_i, z_{i-1})





Mostree: High-level Description

X

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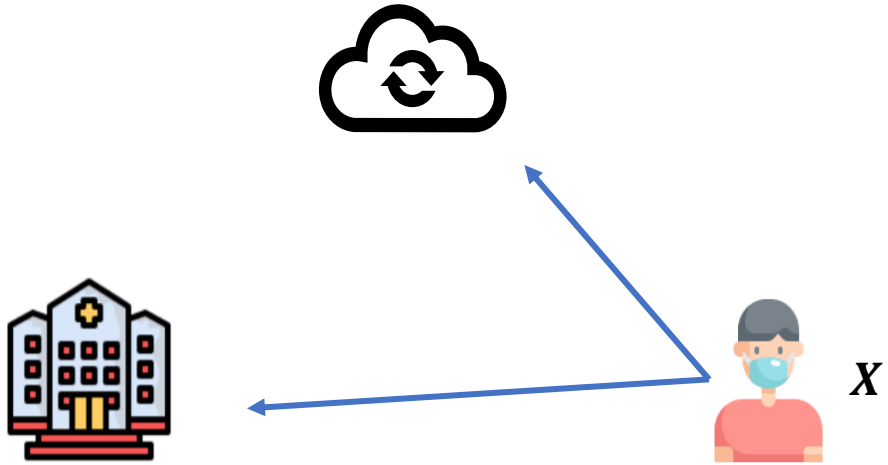

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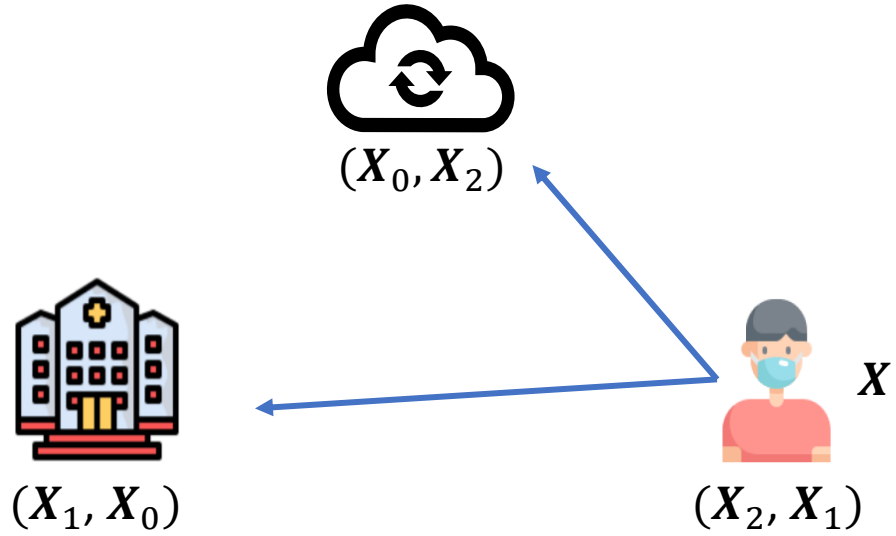

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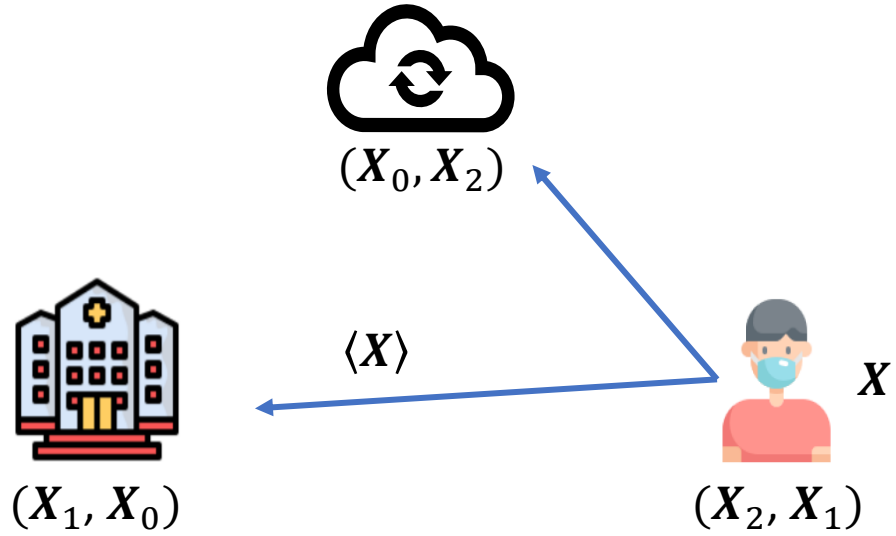


Mostree: High-level Description

(X_0, X_2)

T

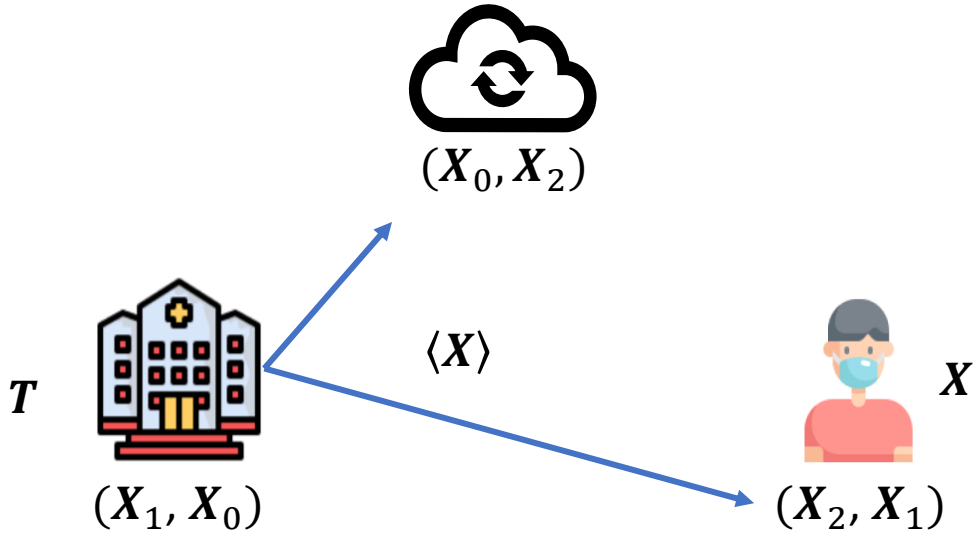
(X_1, X_0)

$\langle X \rangle$

X

(X_2, X_1)

Mostree: High-level Description



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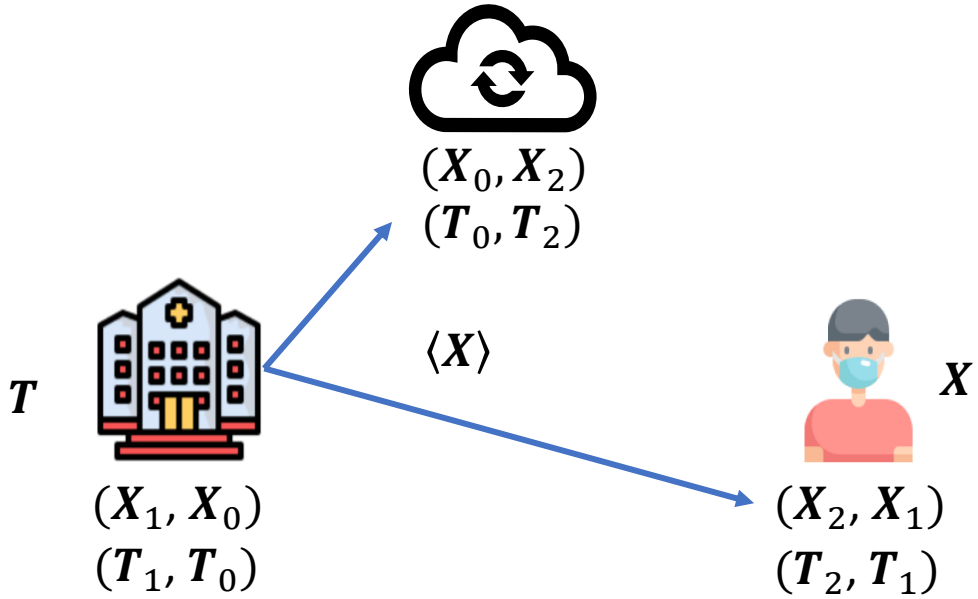

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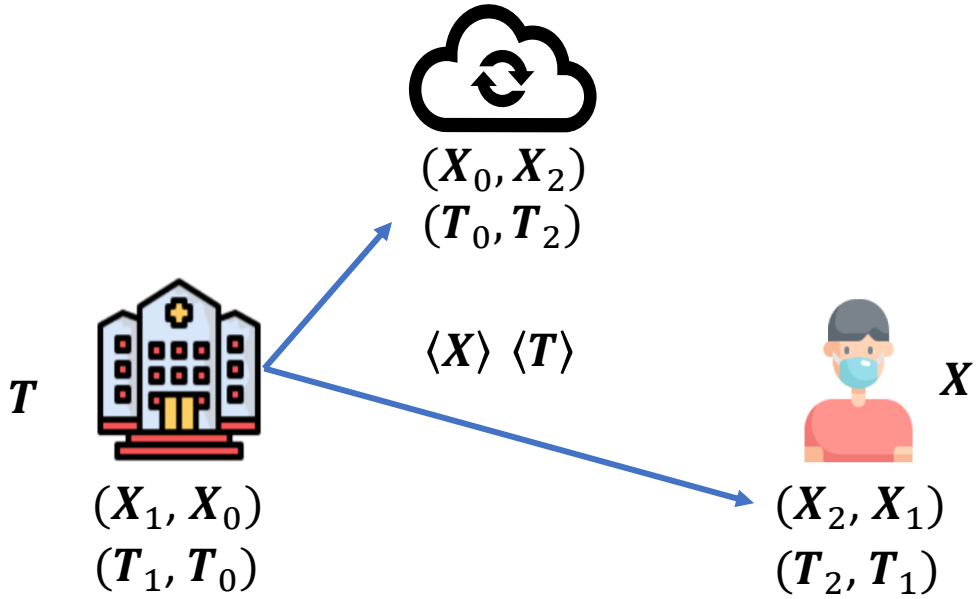

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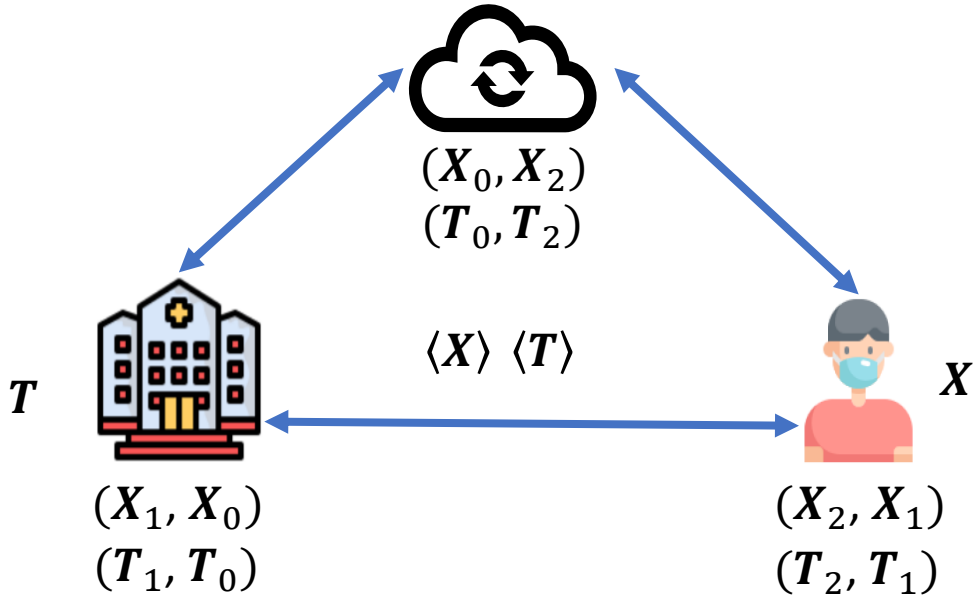

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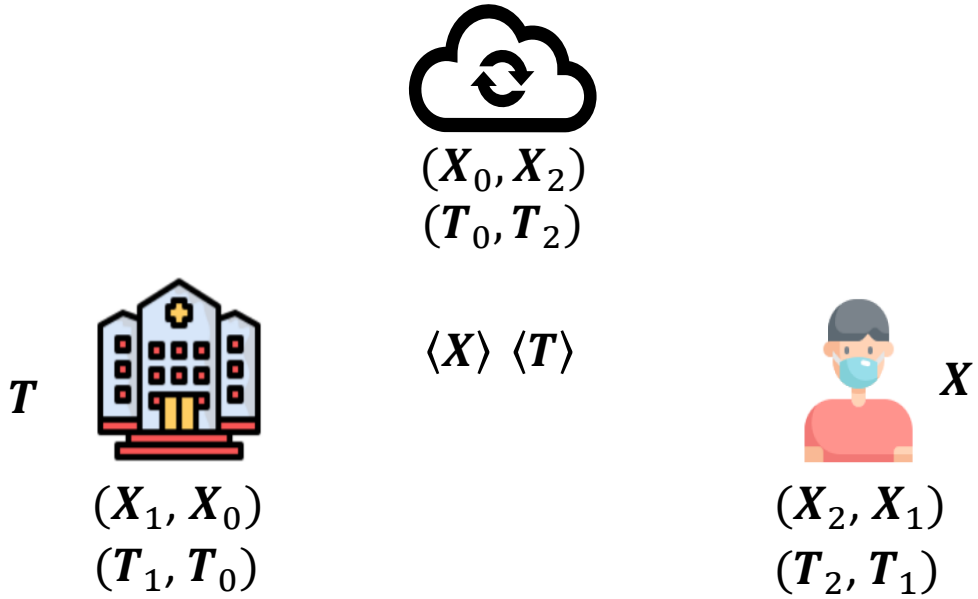

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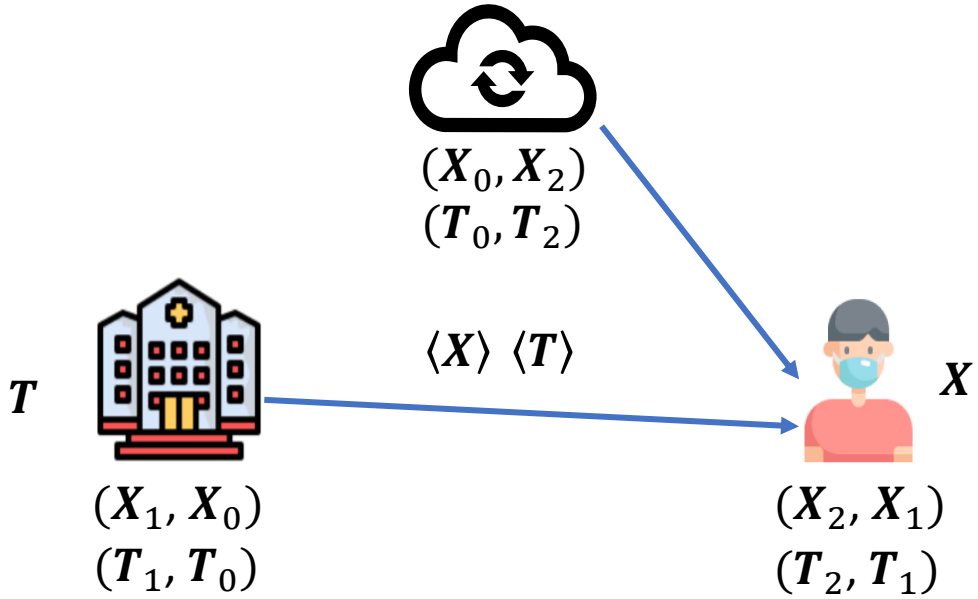

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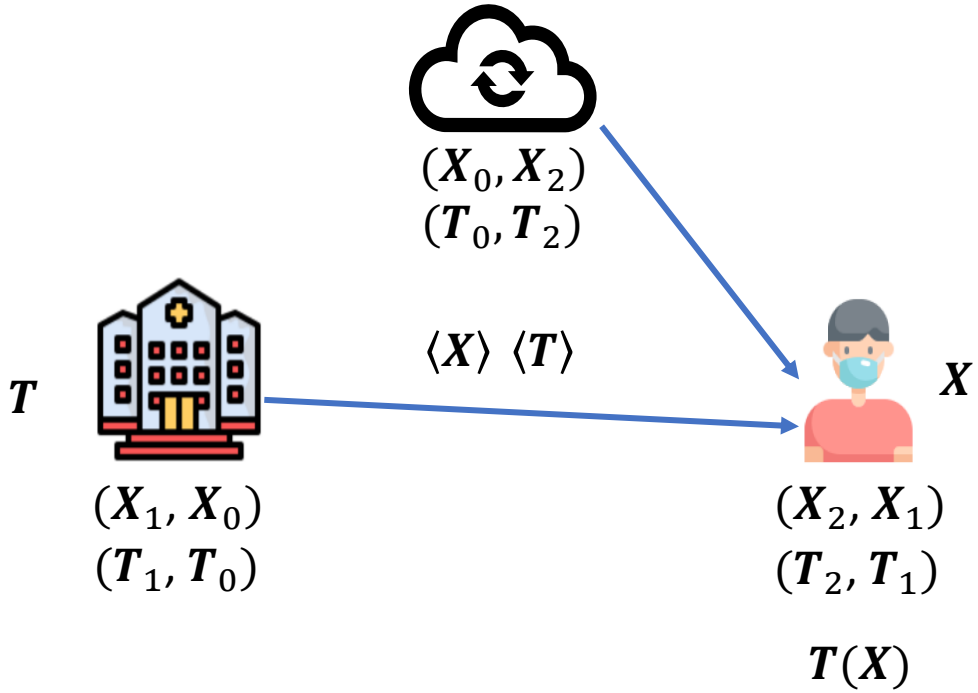
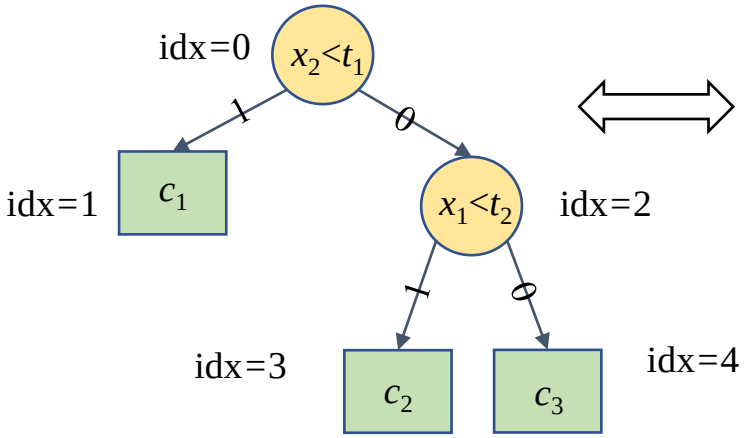


Decision Tree Evaluation



idx	*t*	*l*	*r*	*v*	*c*
0	t_1	1	2	2	⊥
1	⊥	⊥	⊥	⊥	c_1
2	t_2	3	4	1	⊥
3	⊥	⊥	⊥	⊥	c_2
4	⊥	⊥	⊥	⊥	c_3

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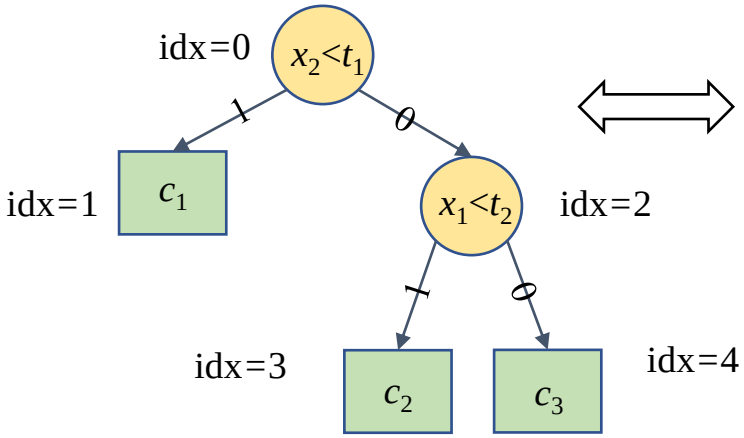

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- Decision tree encoding
  - t: threshold value. $t = \perp$ for decisive nodes.
  - l: index of left child. $l = \perp$ for decisive nodes.
  - r: index of right child. $r = \perp$ for decisive nodes.
  - v: index of attribute to be compared with t.
  - c: classification label. $c = \perp$ for non-leaf nodes.

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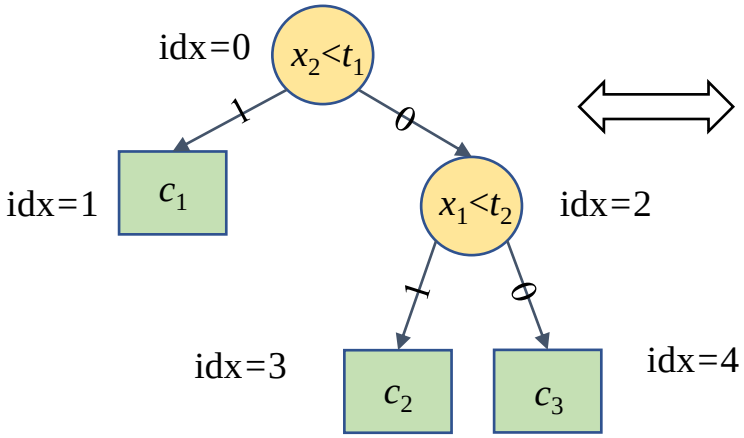

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- $\mathrm{DTE}(T, X)$:
  - $idx \leftarrow 0$
  - For $i \in [1, d]$:
    - $(t, l, r, v, c) \leftarrow T[idx]$
    - If $l = \perp$, return c
    - $x \leftarrow X[v]$
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    - $idx \leftarrow r \oplus b \cdot (r \oplus l)$

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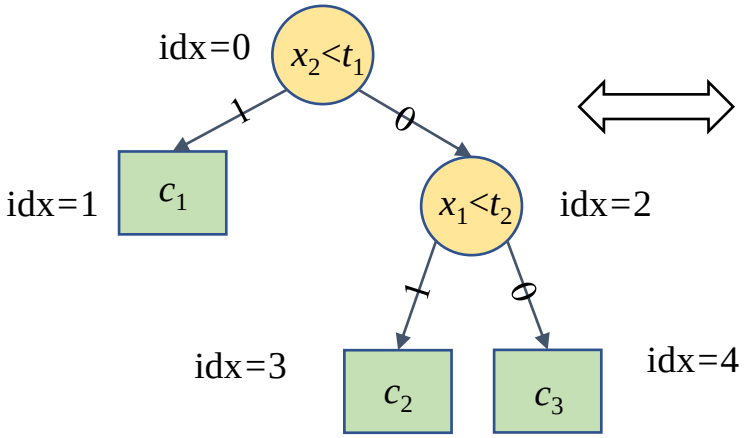

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- Hiding intermediate values
  - Secured by MPC (RSS)

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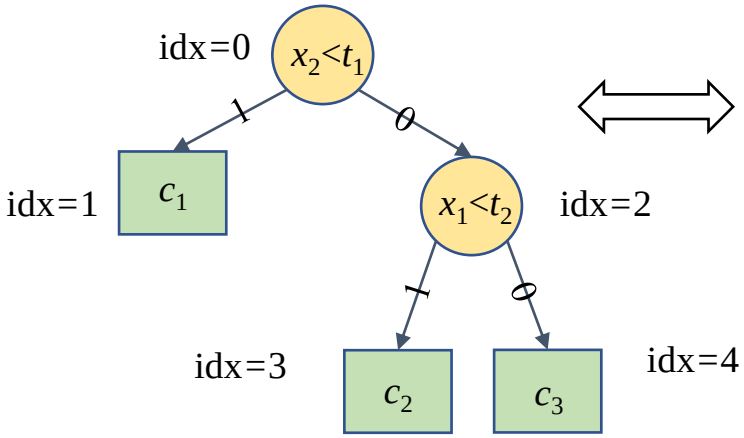

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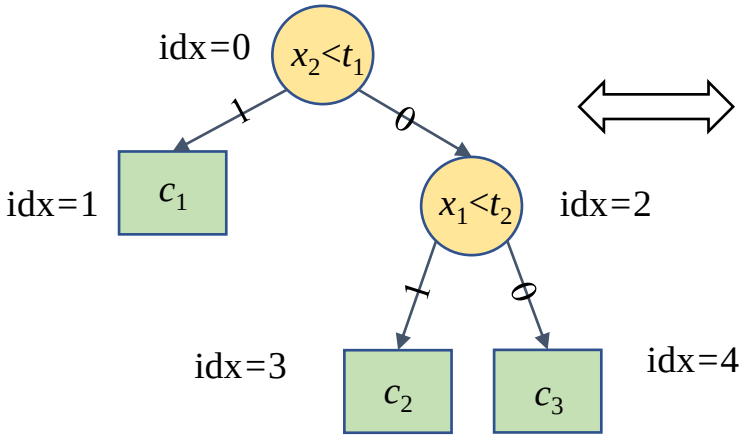

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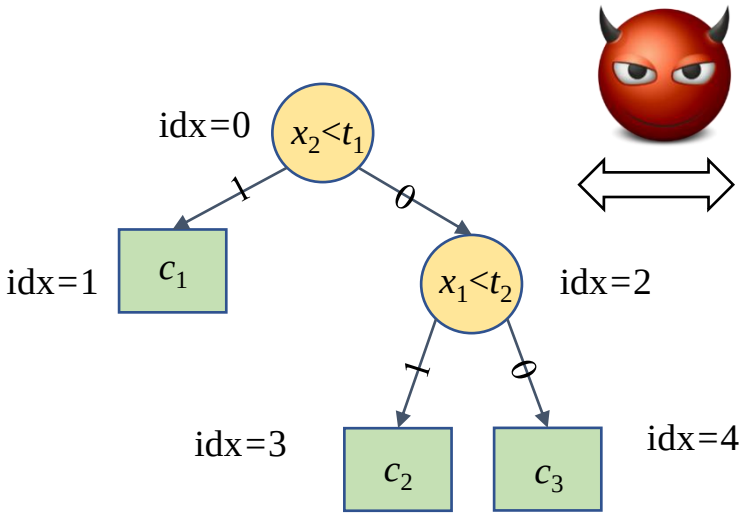

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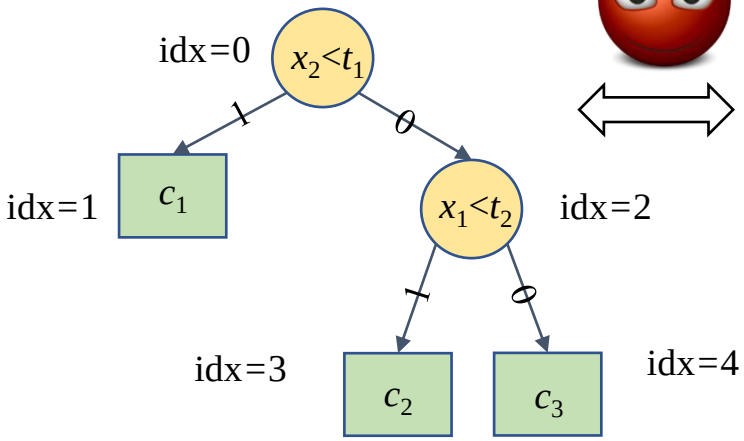

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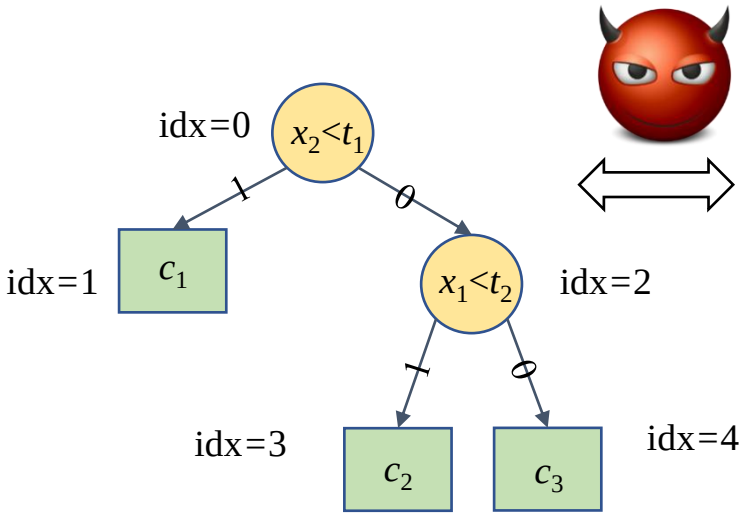

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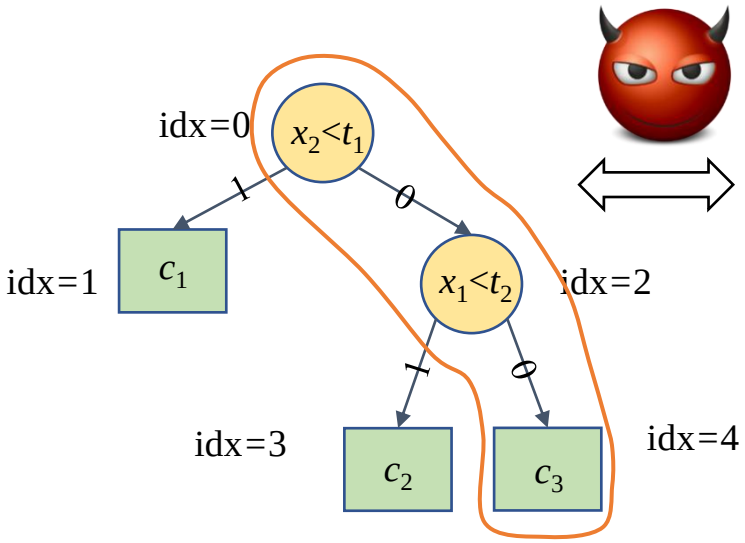

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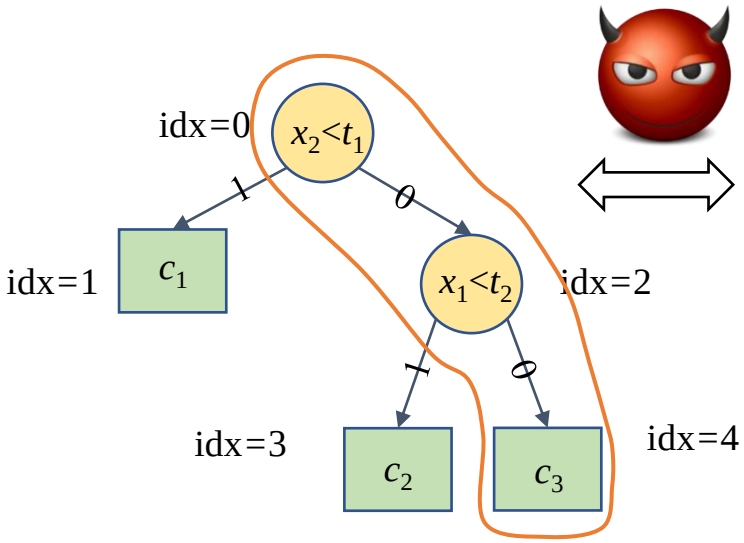

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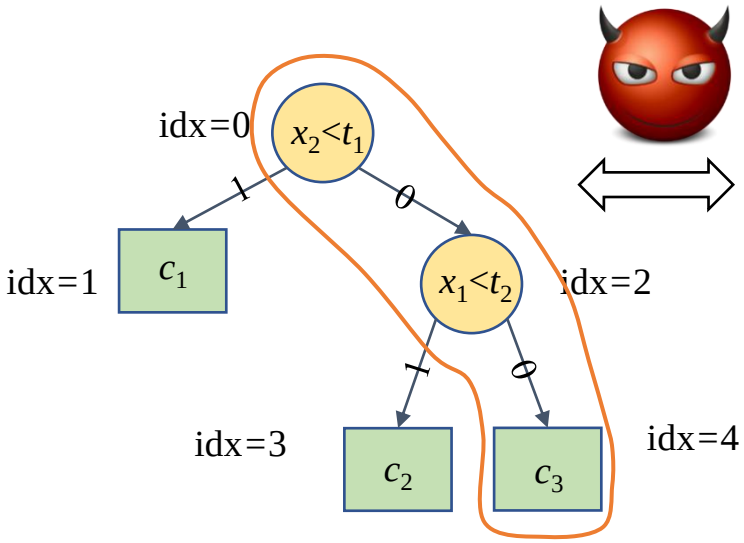
1	$\perp$	$\perp$	$\perp$	$\perp$	c_1
2	t_2	3	4	1	$\perp$
3	$\perp$	$\perp$	$\perp$	$\perp$	c_2
4	$\perp$	$\perp$	$\perp$	$\perp$	c_3

- DTE(T, X):
  - $idx \leftarrow 0$
  - For $i \in [1, d]$:
    - $(t, l, r, v, c) \leftarrow T[idx]$
    - If $l = \perp$, return c
    - $x \leftarrow X[v]$
    - $b \leftarrow x < t$
    - $idx \leftarrow r \oplus b \cdot (r \oplus l)$

- Hiding access pattern
  - Linear scan: linear communication [DEFK19]
  - Oblivious RAM (ORAM):
    - Sublinear communication
    - Heavy overhead: evaluating ORAM circuit within MPC

Decision Tree Evaluation



idx=4

idx	*t*	*l*	*r*	*v*	*c*
0	t_1	1	2	2	$\perp$
1	$\perp$	$\perp$	$\perp$	$\perp$	c_1
2	t_2	3	4	1	$\perp$
3	$\perp$	$\perp$	$\perp$	$\perp$	c_2
4	$\perp$	$\perp$	$\perp$	$\perp$	c_3

- DTE(T, X):
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    - If $l = \perp$, return c
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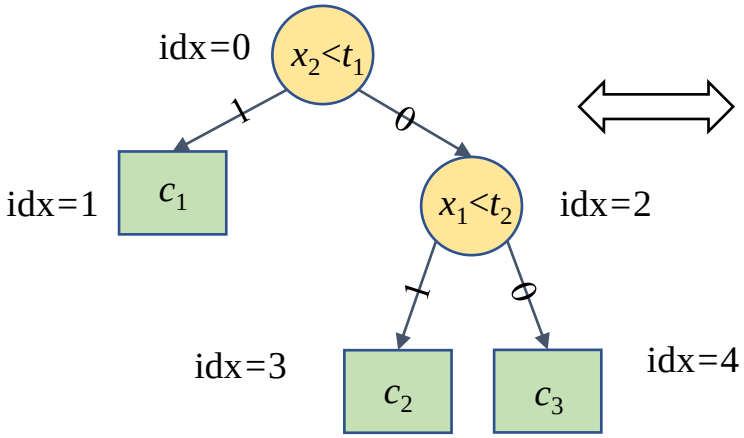
    - $b \leftarrow x < t$
    - $idx \leftarrow r \oplus b \cdot (r \oplus l)$

- Hiding access pattern
  - Linear scan: linear communication [DEFK19]
  - Oblivious RAM (ORAM):
    - Sublinear communication
    - Heavy overhead: evaluating ORAM circuit within MPC
- Observation: DTE only involves read operations, i.e., selection
- Solution: more efficient oblivious selection protocol

Decision Tree Evaluation

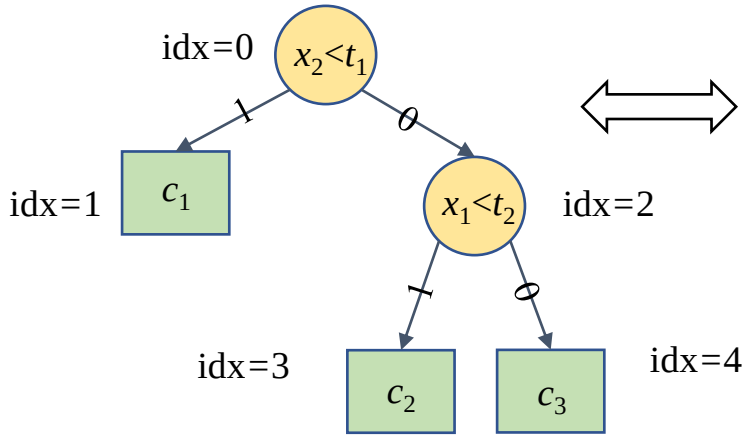


idx	*t*	*l*	*r*	*v*	*c*
0	t_1	1	2	2	$\perp$
1	$\perp$	$\perp$	$\perp$	$\perp$	c_1
2	t_2	3	4	1	$\perp$
3	$\perp$	$\perp$	$\perp$	$\perp$	c_2
4	$\perp$	$\perp$	$\perp$	$\perp$	c_3

- $\mathrm{DTE}(T, X)$:
  - $idx \leftarrow 0$
  - For $i \in [1, d]$:
    - $(t, l, r, v, c) \leftarrow T[idx]$
    - If $l = \perp$, return c
    - $x \leftarrow X[v]$
    - $b \leftarrow x < t$
    - $idx \leftarrow r \oplus b \cdot (r \oplus l)$

- Hiding path length
  - e.g., what if the evaluation return at $i = 2$?
- Solution: dummy evaluation
  - For a decision path with length d, do additional $d' - d$ iterations of dummy evaluation

Decision Tree Evaluation



idx	*t*	*l*	*r*	*v*	*c*
0	t_1	1	2	2	$\perp$
1	$\perp$	$\perp$	$\perp$	$\perp$	c_1
2	t_2	3	4	1	$\perp$
3	$\perp$	$\perp$	$\perp$	$\perp$	c_2
4	$\perp$	$\perp$	$\perp$	$\perp$	c_3

- DTE(T, X):
  - $idx \leftarrow 0$
  - For $i \in [1, d]$:
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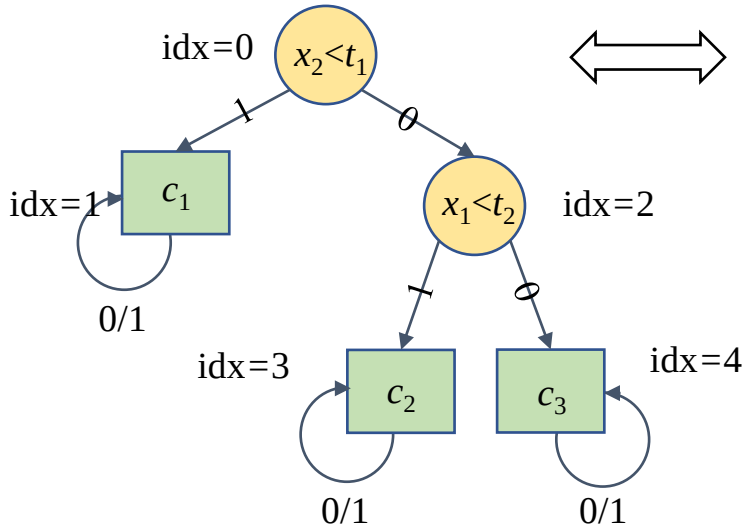
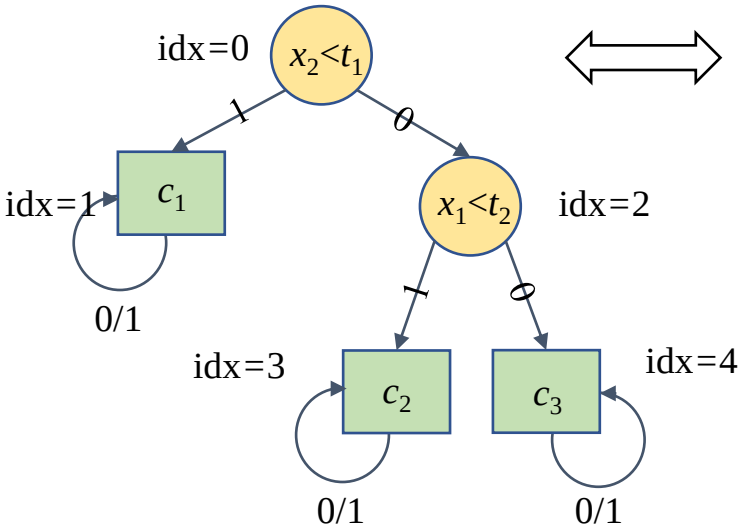


idx	*t*	*l*	*r*	*v*	*c*
0	t_1	1	2	2	$\perp$
1	$\perp$	1	1	$\perp$	c_1
2	t_2	3	4	1	$\perp$
3	$\perp$	3	3	$\perp$	c_2
4	$\perp$	4	4	$\perp$	c_3

- DTE(T, X):
  - $idx \leftarrow 0, result \leftarrow \perp$
  - For $i \in [1, d']$: $//d' \geq d$
    - $(t, l, r, v, c) \leftarrow T[idx]$
    - $x \leftarrow X[v]$
    - $b \leftarrow x < t$
    - $idx \leftarrow r \oplus b \cdot (r \oplus l)$
    - $result \leftarrow c$
  - Return $result$

Private Decision Tree Evaluation



idx	*t*	*l*	*r*	*v*	*c*
0	t_1	1	2	2	$\perp$
1	$\perp$	1	1	$\perp$	c_1
2	t_2	3	4	1	$\perp$
3	$\perp$	3	3	$\perp$	c_2
4	$\perp$	4	4	$\perp$	c_3

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    - $x \leftarrow X[v]$
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    - $result \leftarrow c$
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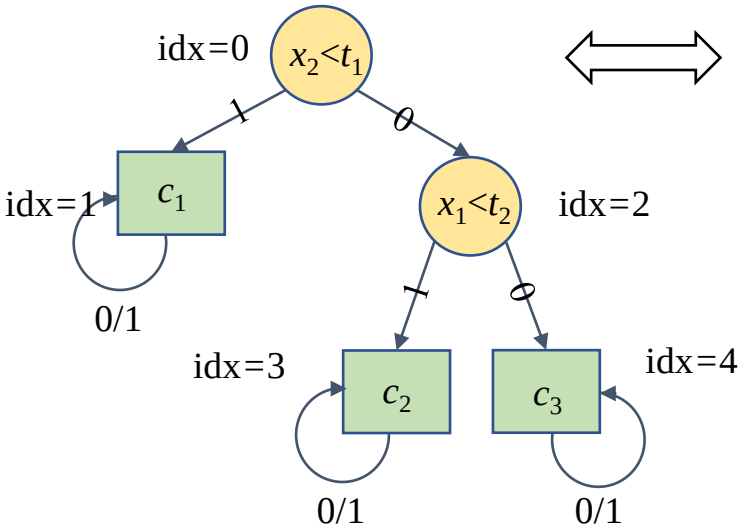
- Hiding sensitive values of tree/feature
  - via MPC
- Hiding access pattern
  - via oblivious selection protocols
- Hiding length information
  - via dummy evaluation

Private Decision Tree Evaluation



idx	*t*	*l*	*r*	*v*	*c*
0	t_1	1	2	2	$\perp$
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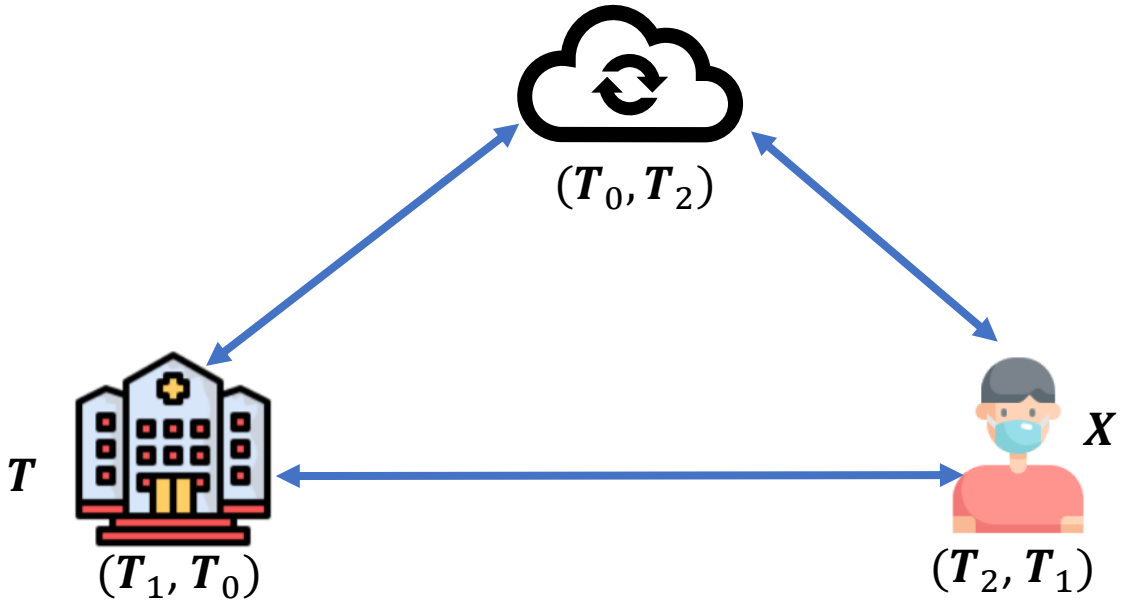
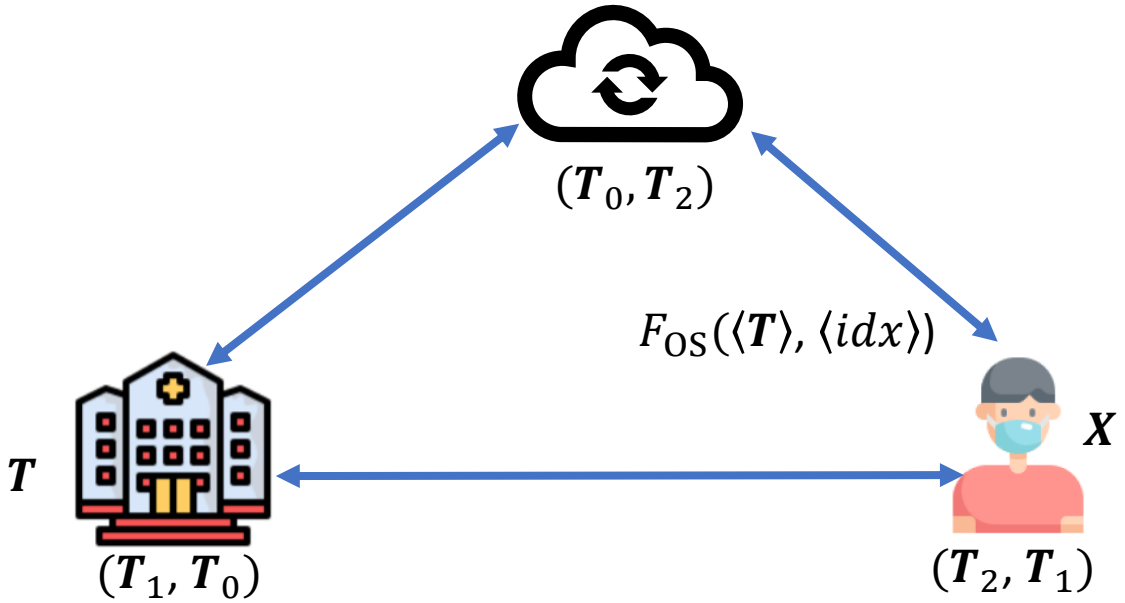
Three-party Oblivious Selection (OS)



- Security
  - Privacy: No party knows idx nor $\boldsymbol{T}[idx]$
  - Correctness: The parties share the correct selection result $\boldsymbol{T}[idx]$
- Efficiency
  - Sublinear communication (in $|\boldsymbol{T}|$)

Three-party Oblivious Selection (OS)



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  - Privacy: No party knows idx nor $T[idx]$
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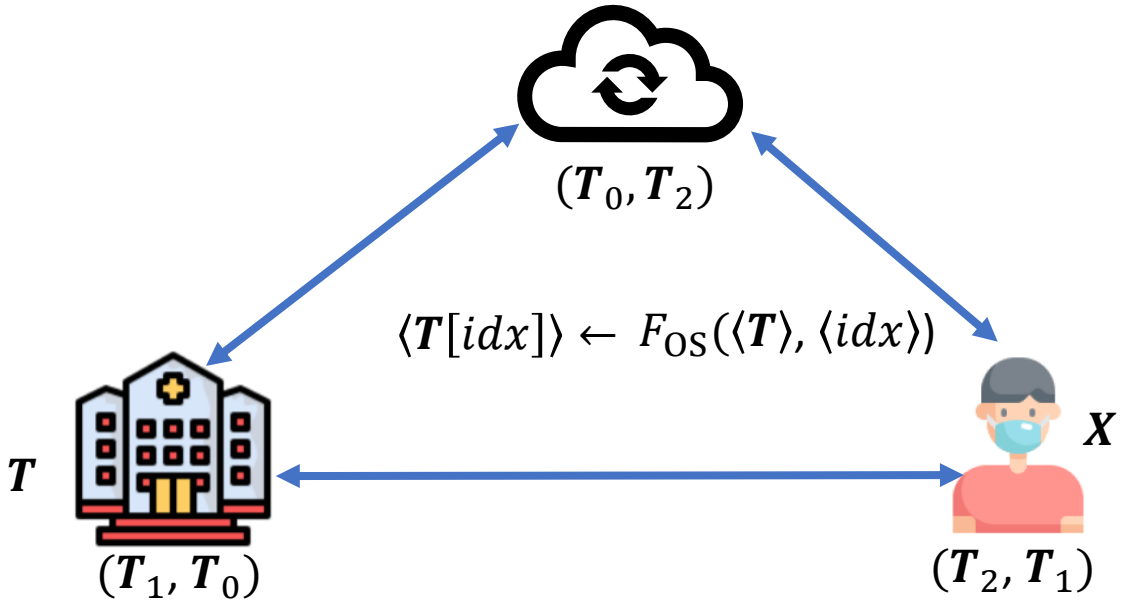

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  - Privacy: No party knows idx nor $T[idx]$
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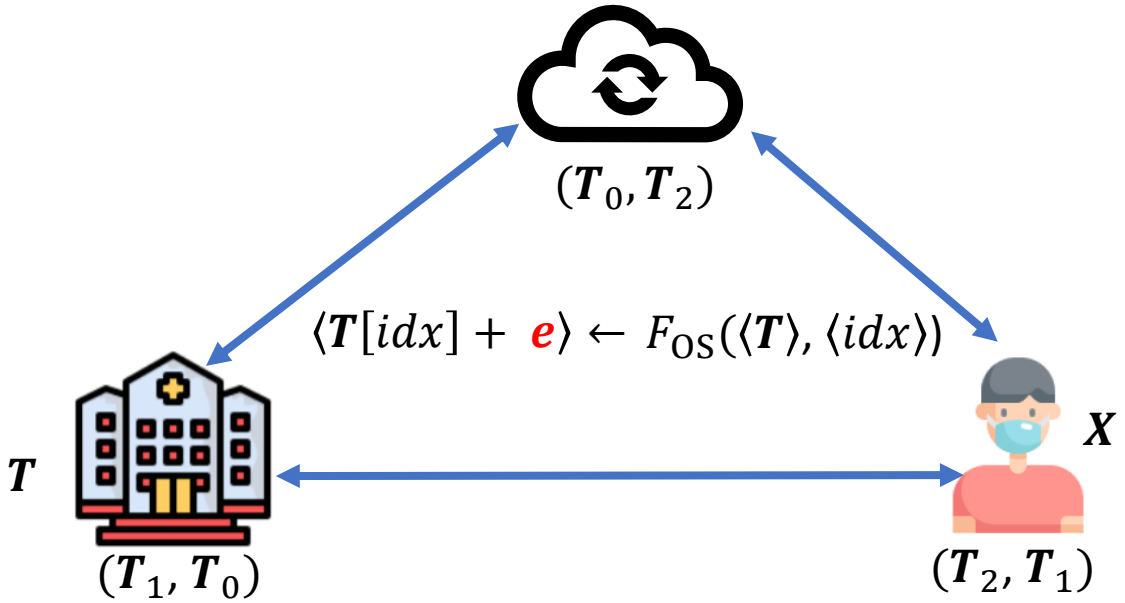


- Security
  - Privacy: No party knows idx nor $T[idx]$ ☺
  - Correctness: The adversary can add an error e to the result ☹
- Efficiency
  - Sublinear communication (in $|T|$)

3PC Oblivious Selection using Pure RSS

Oblivious Selection from Pure RSS

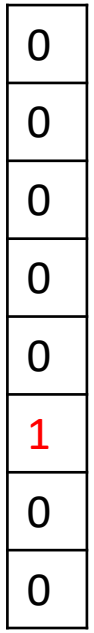
$$\boldsymbol{v}[i] = \begin{cases} 0, i \neq idx \\ 1, i = idx \end{cases}$$

- OS from inner-product computation.
  - Input: Vector sharing $\langle \boldsymbol{T} \rangle$; one-hot RSS-shared vector $(\langle \boldsymbol{v} \rangle, \langle idx \rangle)$.
  - Output: $\langle \boldsymbol{T}[idx] \rangle$.

- Security/Efficiency issues:
  1. $O(n)$ communication for generating $\langle \boldsymbol{v} \rangle$.
  2. $O(n)$ communication from n secret-shared multiplications.
  3. Additive attacks from a malicious party.

Derandomization for One-hot Vector

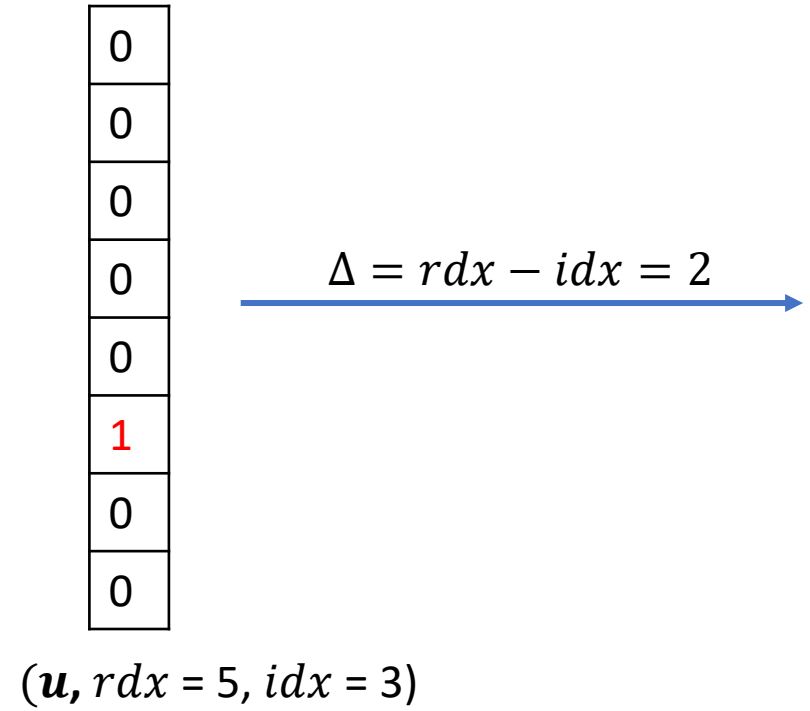
- Suppose the parties already share a one-hot vector $(\langle \boldsymbol{u} \rangle, \langle rdx \rangle)$



$(\boldsymbol{u}, rdx = 5, idx = 3)$

Derandomization for One-hot Vector

- Suppose the parties already share a one-hot vector $(\langle \boldsymbol{u} \rangle, \langle rdx \rangle)$



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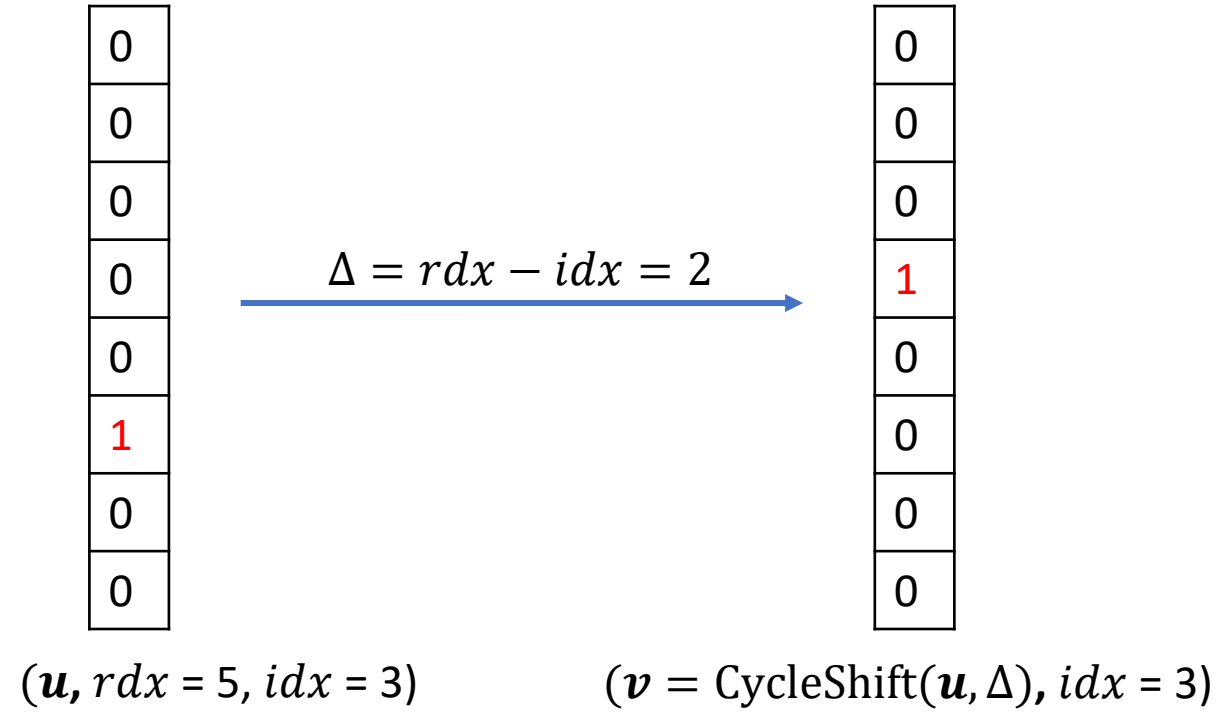

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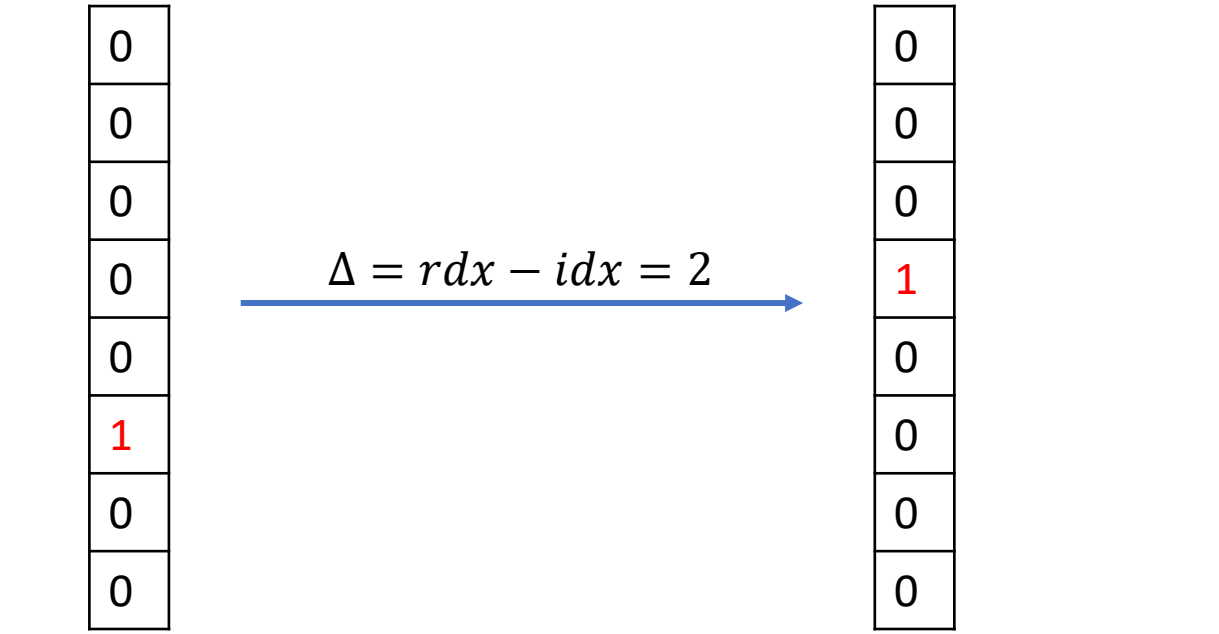


$(\boldsymbol{u}, rdx = 5, idx = 3)$ $(\boldsymbol{v} = \text{CycleShift}(\boldsymbol{u}, \Delta), idx = 3)$

$$\boldsymbol{v}[i] = \boldsymbol{u}[i + \Delta] = \begin{cases} 1, i = idx \\ 0, i \neq idx \end{cases}$$

Oblivious Selection with up to Additive Attacks

- Check correctness of multiplication:
  - Previous approach: Using an RSS triple $(\langle a\rangle, \langle b\rangle, \langle ab\rangle)$ to check correctness
  - $O(n)$ communication in total

Oblivious Selection with up to Additive Attacks

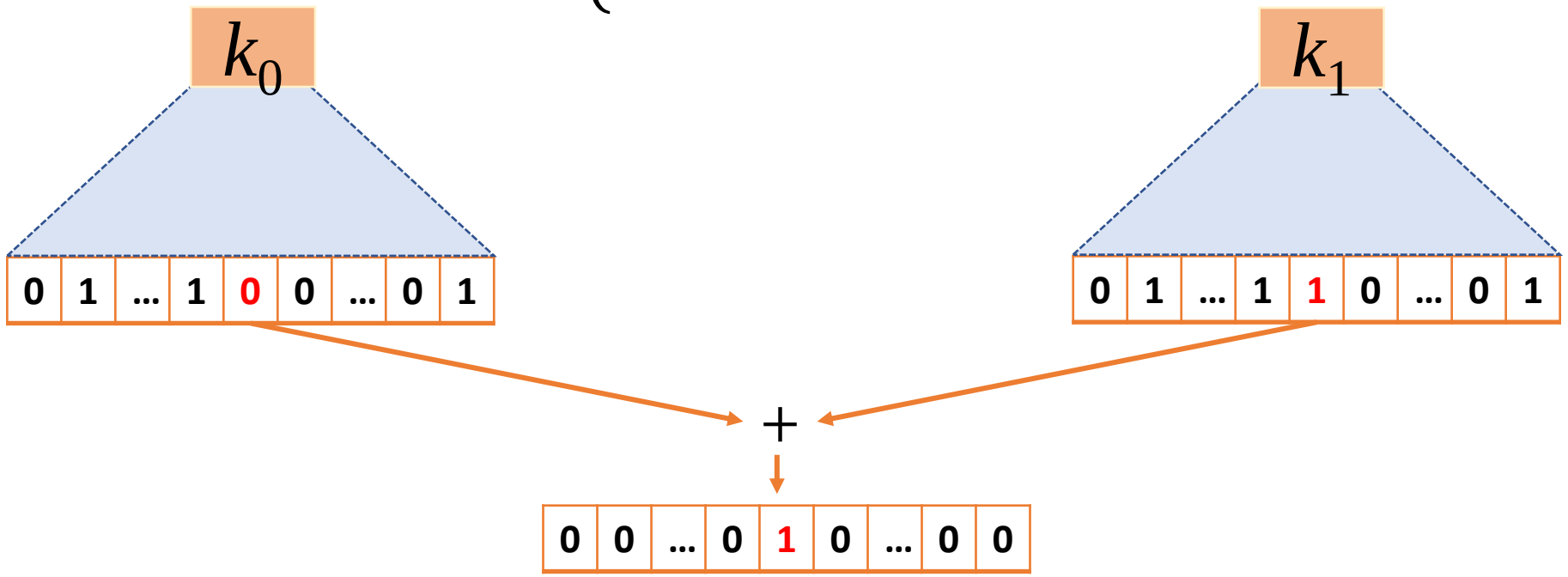
- Check correctness of multiplication:
  - Previous approach: Using an RSS triple $(\langle a\rangle, \langle b\rangle, \langle ab\rangle)$ to check correctness
  - $O(n)$ communication in total

- Our method
  - No correctness check for multiplication
  - Up to additive attacks
  - $O(1)$ communication for resharing

3PC Oblivious Selection using Distributed Point Functions

Tool: Distributed Point Function (DPF)

- A point function $f_{\alpha,\beta}(x) = \begin{cases} \beta, & \text{if } x = \alpha, \\ 0, & \text{otherwise.} \end{cases}$



Oblivious Selection from RSS+DPF

$(\boldsymbol{T}_0, \boldsymbol{T}_2)$

$(\boldsymbol{T}_1, \boldsymbol{T}_0)$

$(\boldsymbol{T}_2, \boldsymbol{T}_1)$

Oblivious Selection from RSS+DPF

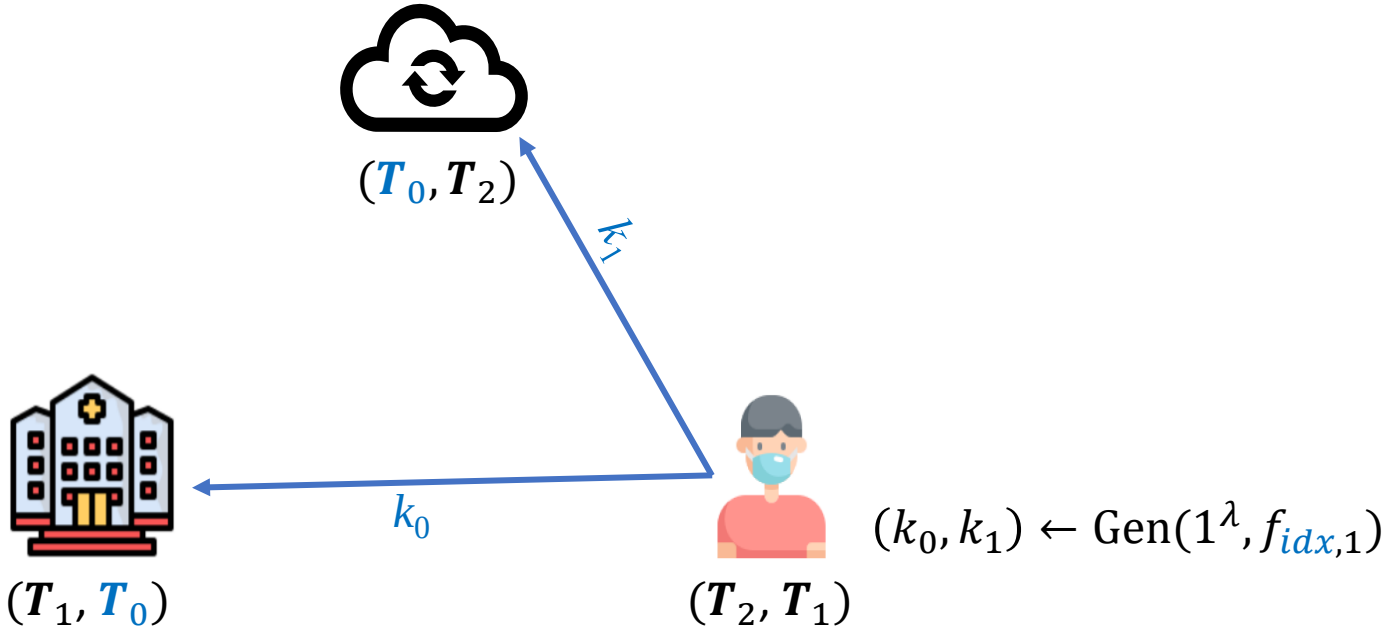
$(\boldsymbol{T}_0, \boldsymbol{T}_2)$

$(\boldsymbol{T}_1, \boldsymbol{T}_0)$

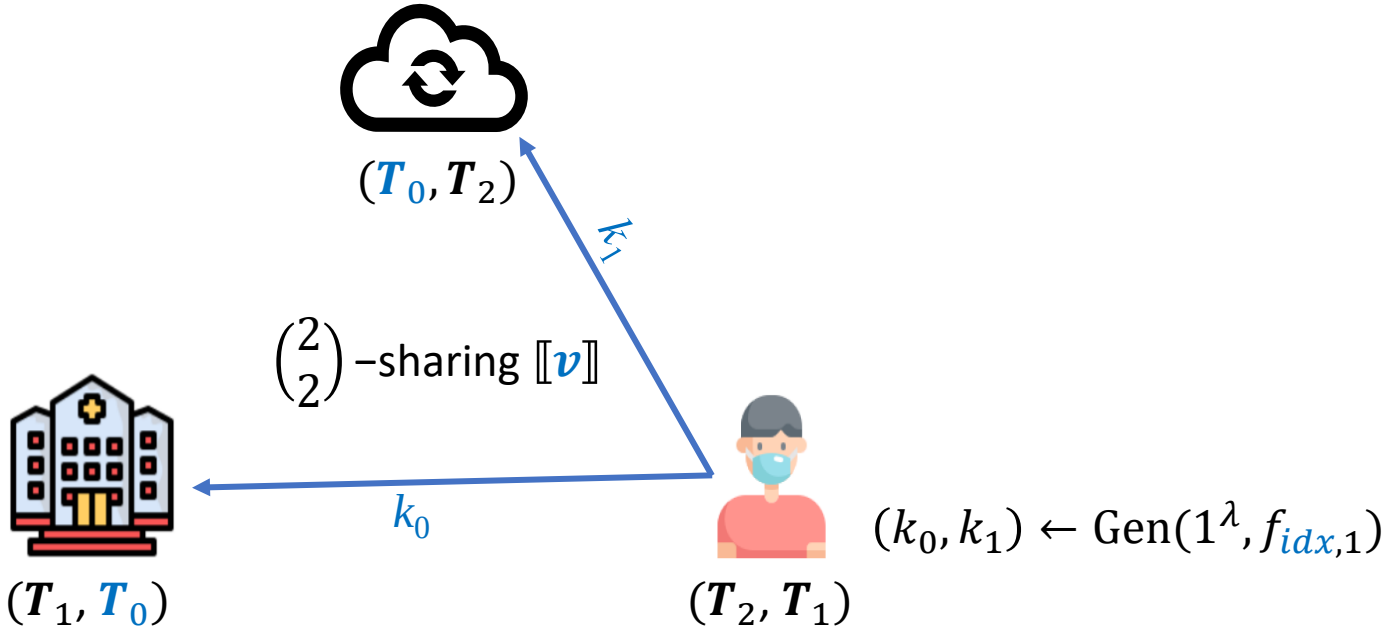
$(\boldsymbol{T}_2, \boldsymbol{T}_1)$

$(k_0, k_1) \leftarrow \mathrm{Gen}(1^\lambda, f_{idx,1})$

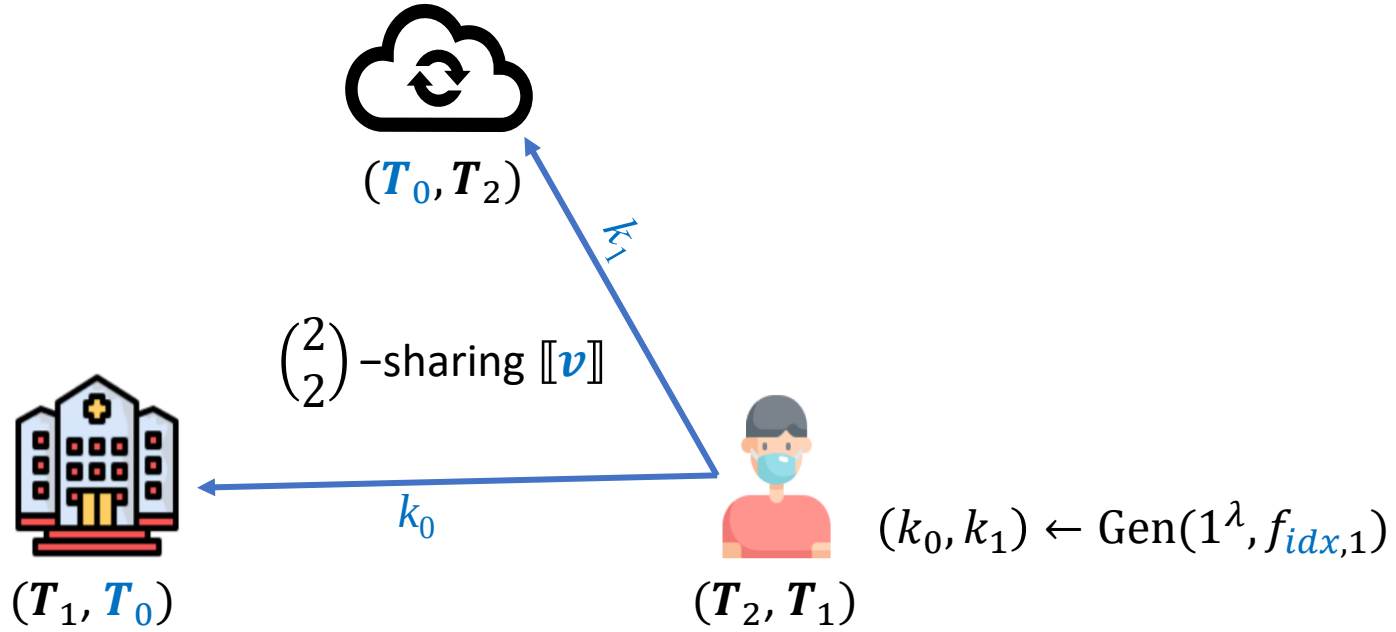
Oblivious Selection from RSS+DPF



Oblivious Selection from RSS+DPF

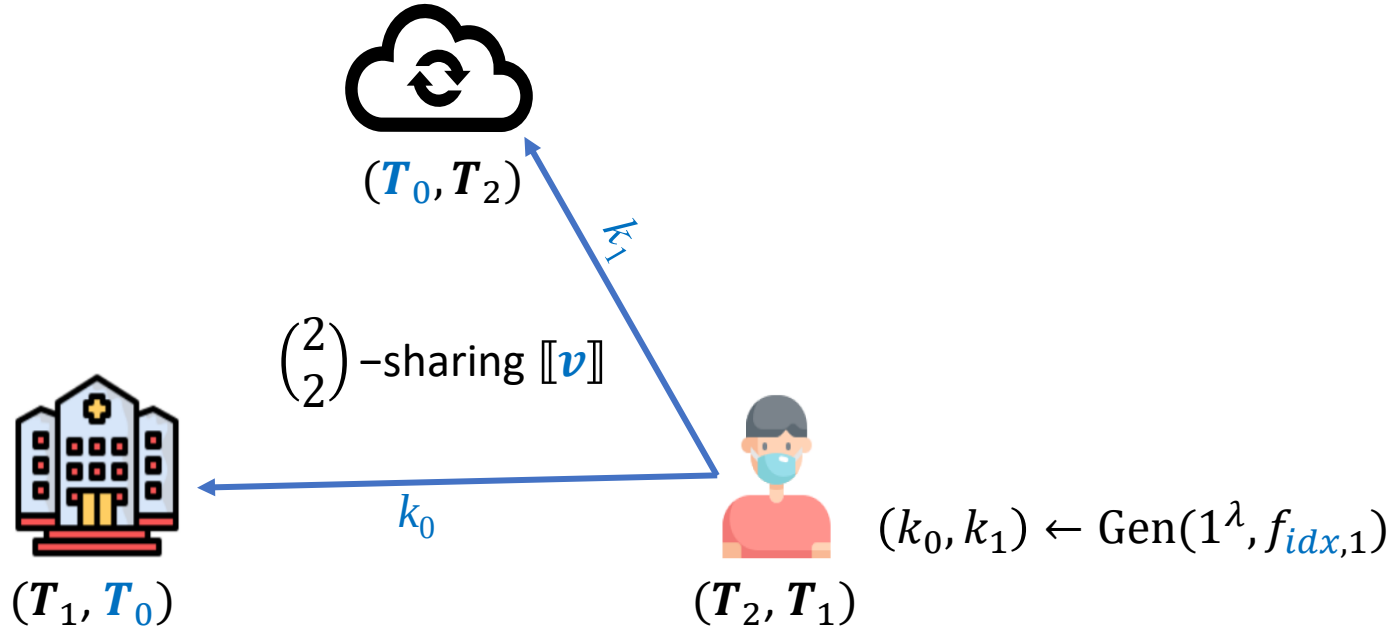


Oblivious Selection from RSS+DPF



- Two-party inner-product computation
  - Input: $[\![\boldsymbol{v}]\!]$ and $\boldsymbol{T}_0$
  - Output: $\binom{2}{2}$–sharing $[\![\boldsymbol{T}_0[idx]]\!]$

Oblivious Selection from RSS+DPF



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  - Input: $[\![\boldsymbol{v}]\!]$ and $\boldsymbol{T}_0$
  - Output: $\binom{2}{2}$–sharing $[\![\boldsymbol{T}_0[idx]]\!]$
- Caveat: The dealer learns idx

Oblivious Selection from RSS+DPF

$(\boldsymbol{T}_0, \boldsymbol{T}_2)$

$(\boldsymbol{T}_1, \boldsymbol{T}_0)$

$(\boldsymbol{T}_2, \boldsymbol{T}_1)$

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  - Input: $[\![\boldsymbol{v}]\!]$ and $\boldsymbol{T}_0$
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- Caveat: The dealer learns idx
  - Solution: Derandomization for one-hot vector

Oblivious Selection from RSS+DPF

$(\boldsymbol{T}_0, \boldsymbol{T}_2)$

$(\boldsymbol{T}_1, \boldsymbol{T}_0)$

$(\boldsymbol{T}_2, \boldsymbol{T}_1)$

$(k_0, k_1) \leftarrow \mathrm{Gen}(f_{rdx,1})$

- Two-party inner-product computation
  - Input: $[\![\boldsymbol{v}]\!]$ and $\boldsymbol{T}_0$
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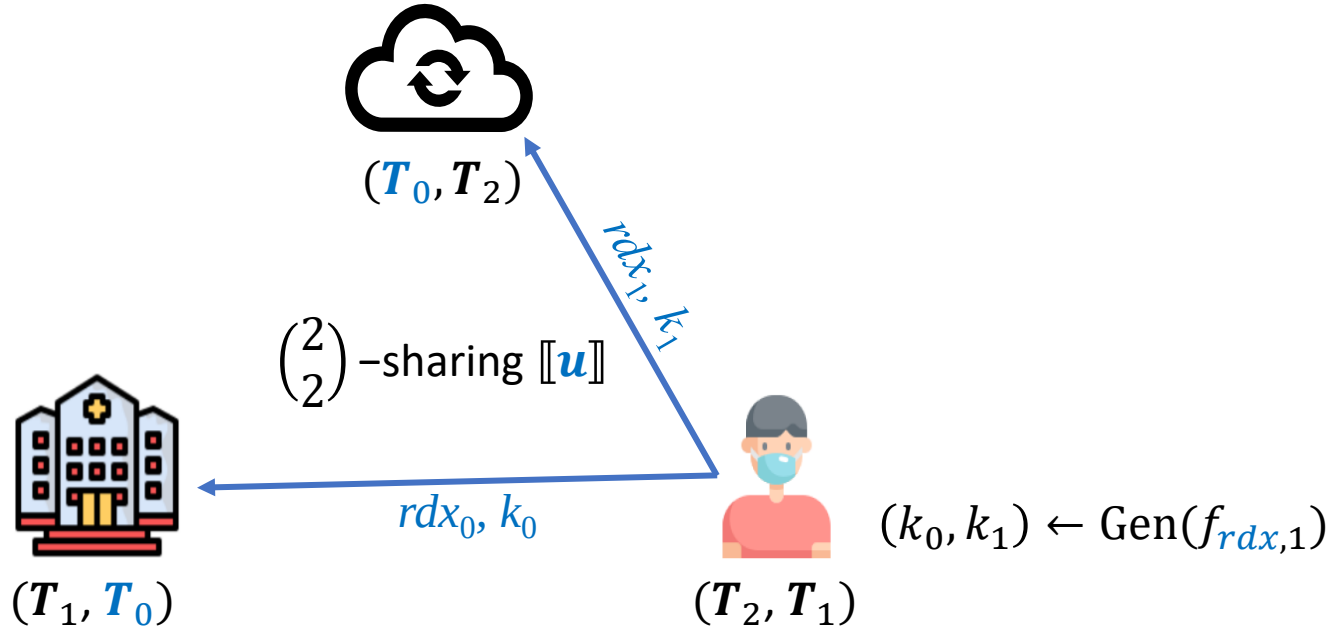
Oblivious Selection from RSS+DPF



- Two-party inner-product computation
  - Input: $[\![\boldsymbol{v}]\!]$ and $\boldsymbol{T}_0$
  - Output: $\binom{2}{2}$–sharing $[\![\boldsymbol{T}_0[idx]]\!]$
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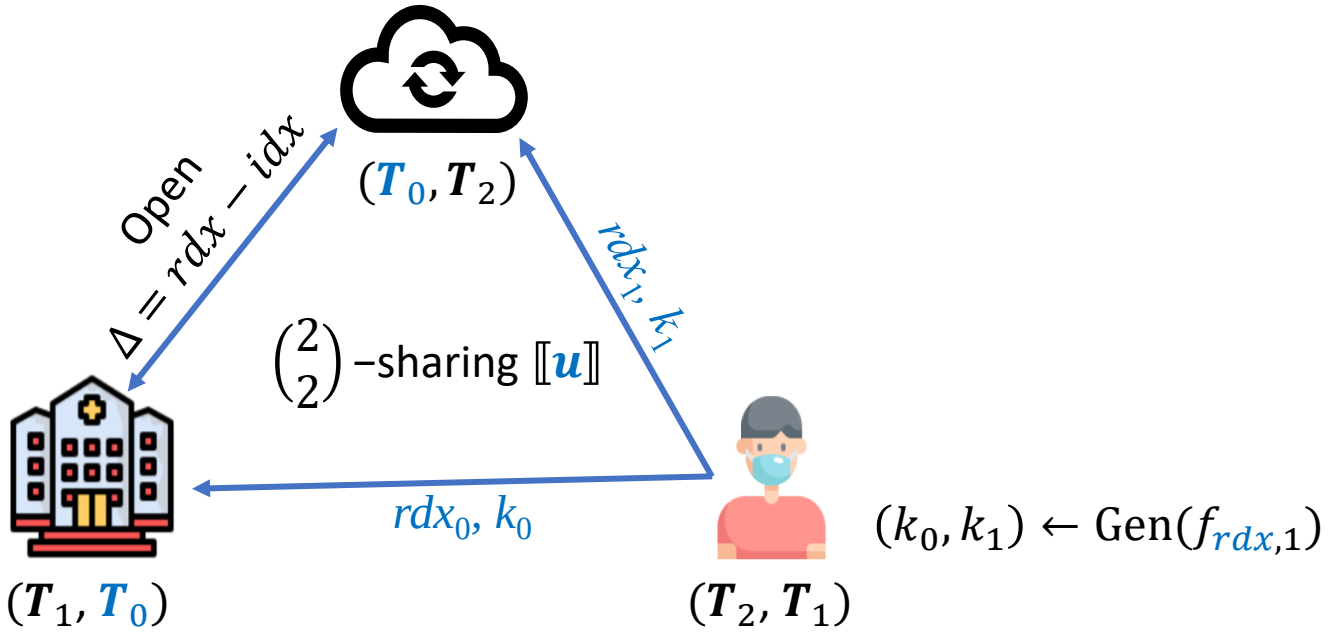
Oblivious Selection from RSS+DPF



- Two-party inner-product computation
  - Input: $[\![\boldsymbol{v}]\!]$ and $\boldsymbol{T}_0$
  - Output: $\binom{2}{2}$–sharing $[\![\boldsymbol{T}_0[idx]]\!]$
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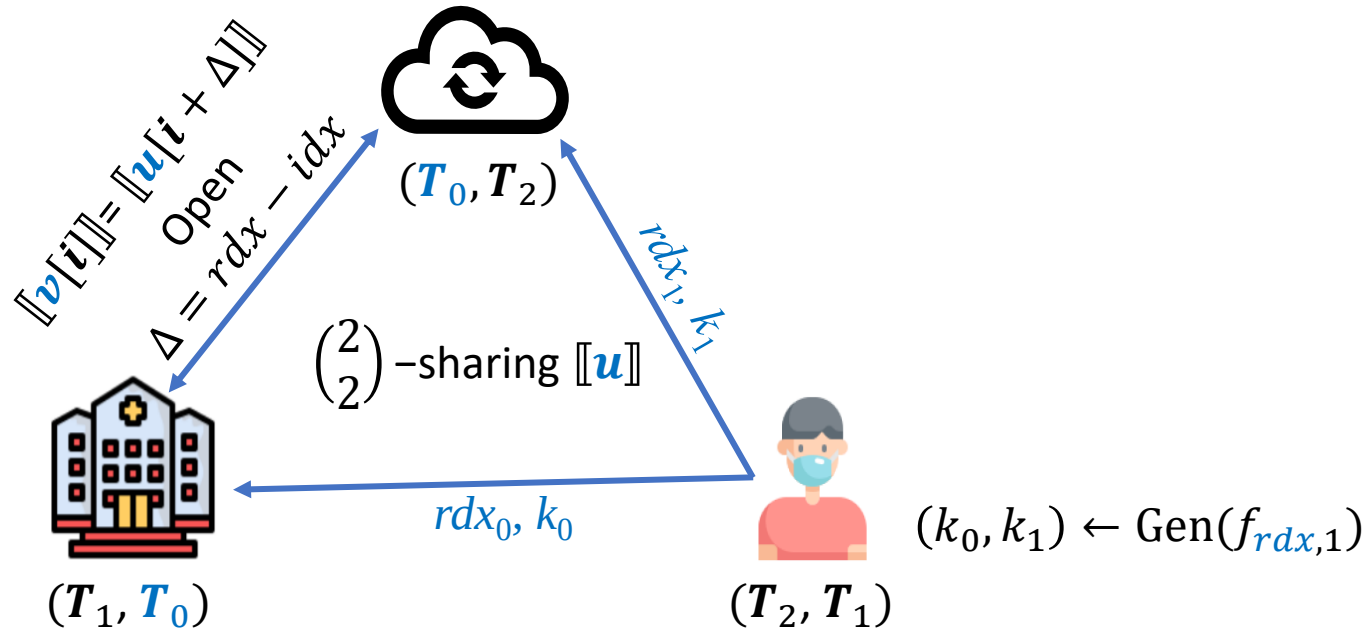
Oblivious Selection from RSS+DPF



- Two-party inner-product computation
  - Input: $[\![v]\!]$ and T_0
  - Output: $\binom{2}{2}$−sharing $[\![T_0[idx]]\!]$
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Oblivious Selection from RSS+DPF

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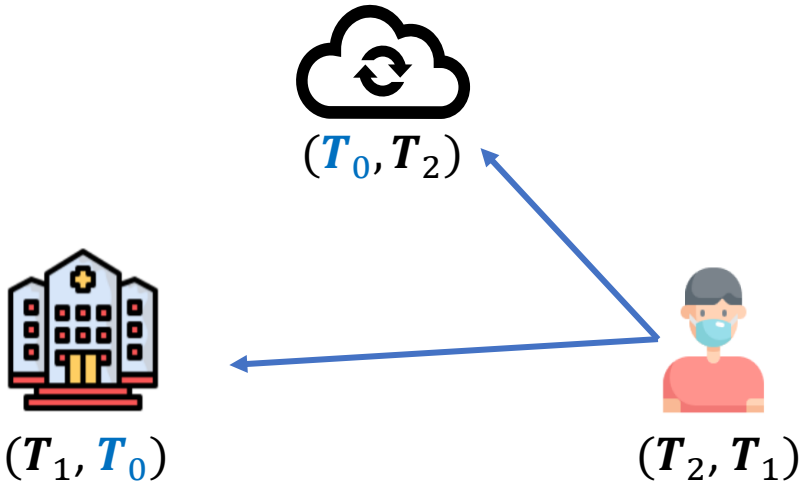
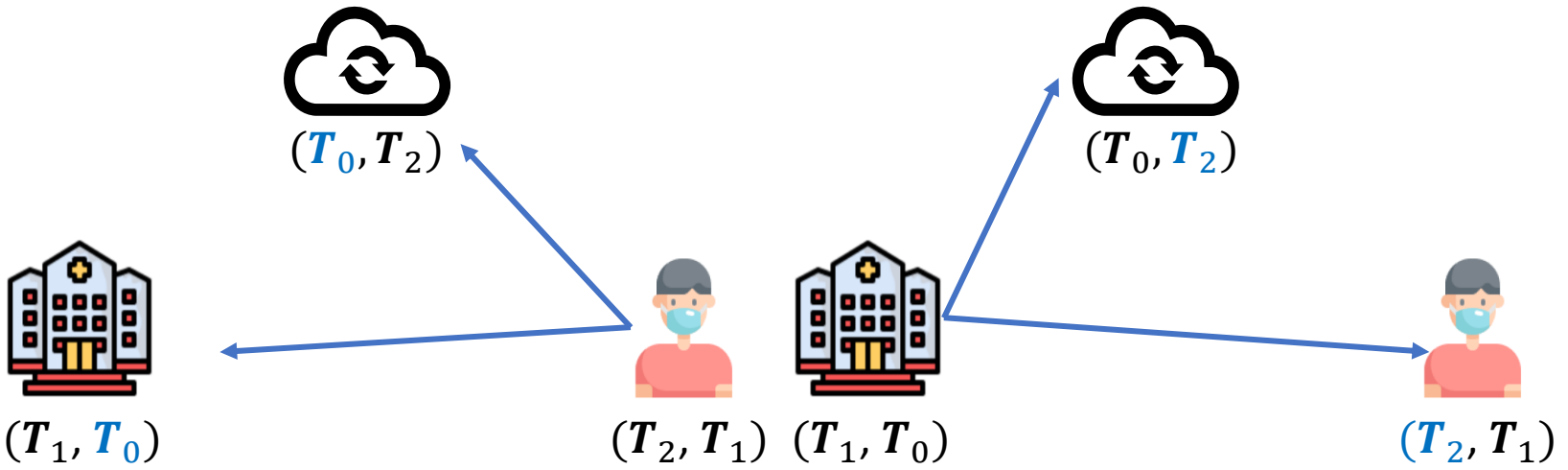


Oblivious Selection from RSS+DPF



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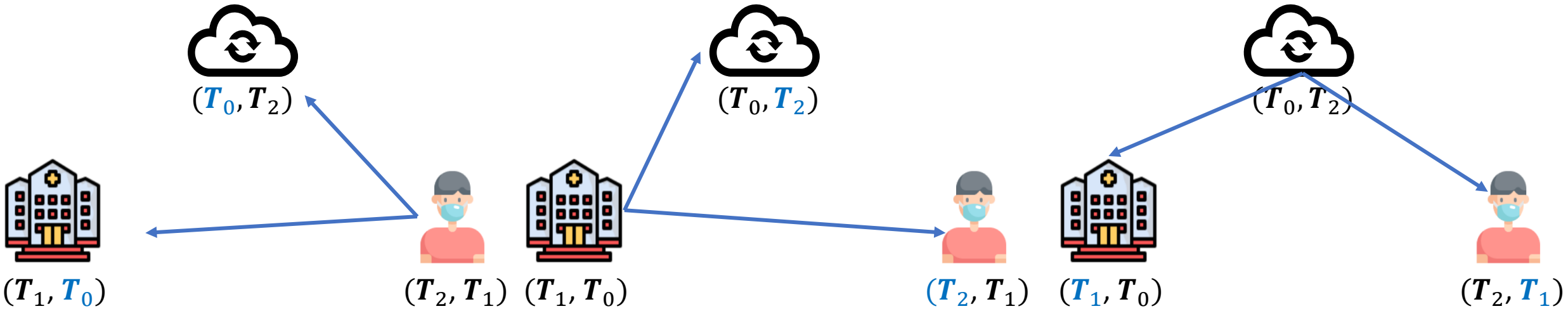

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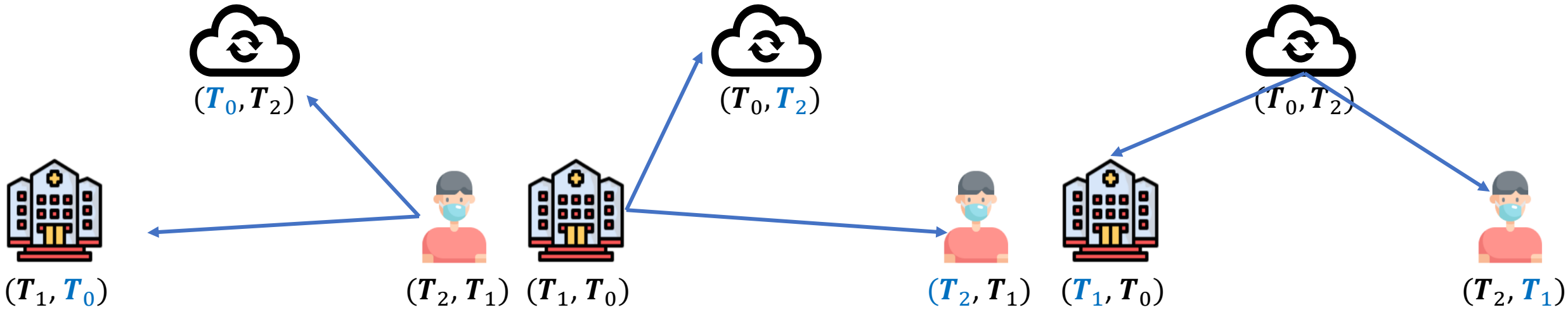
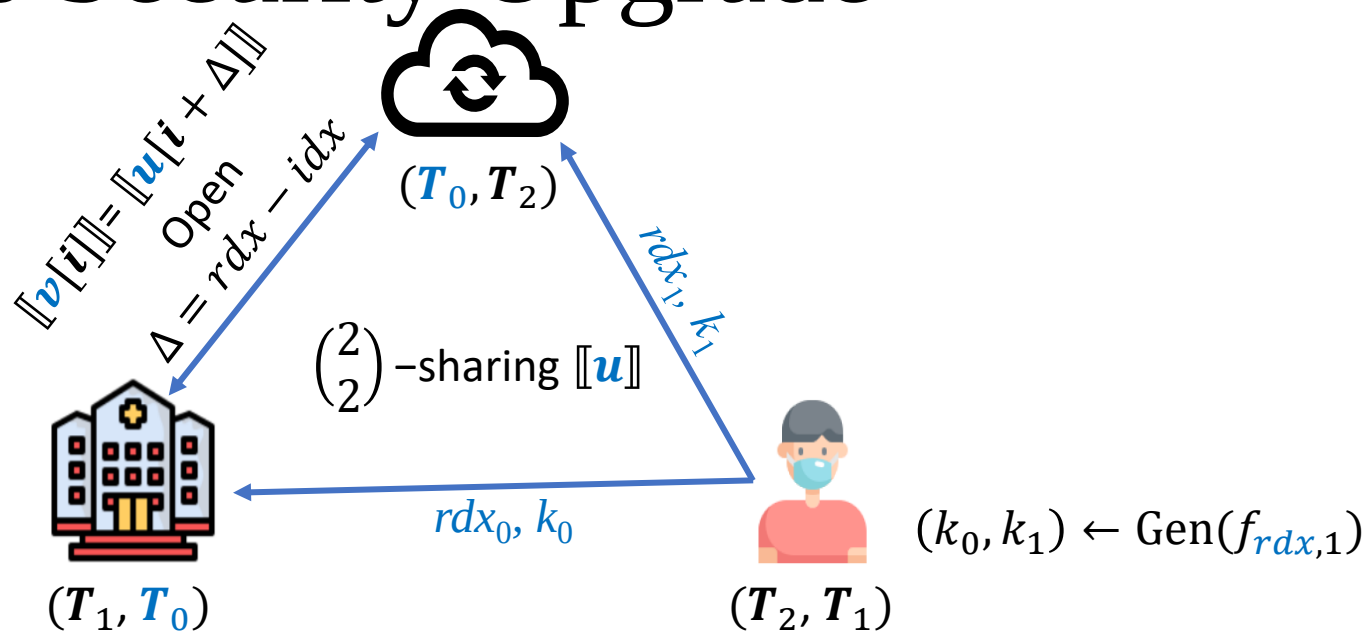


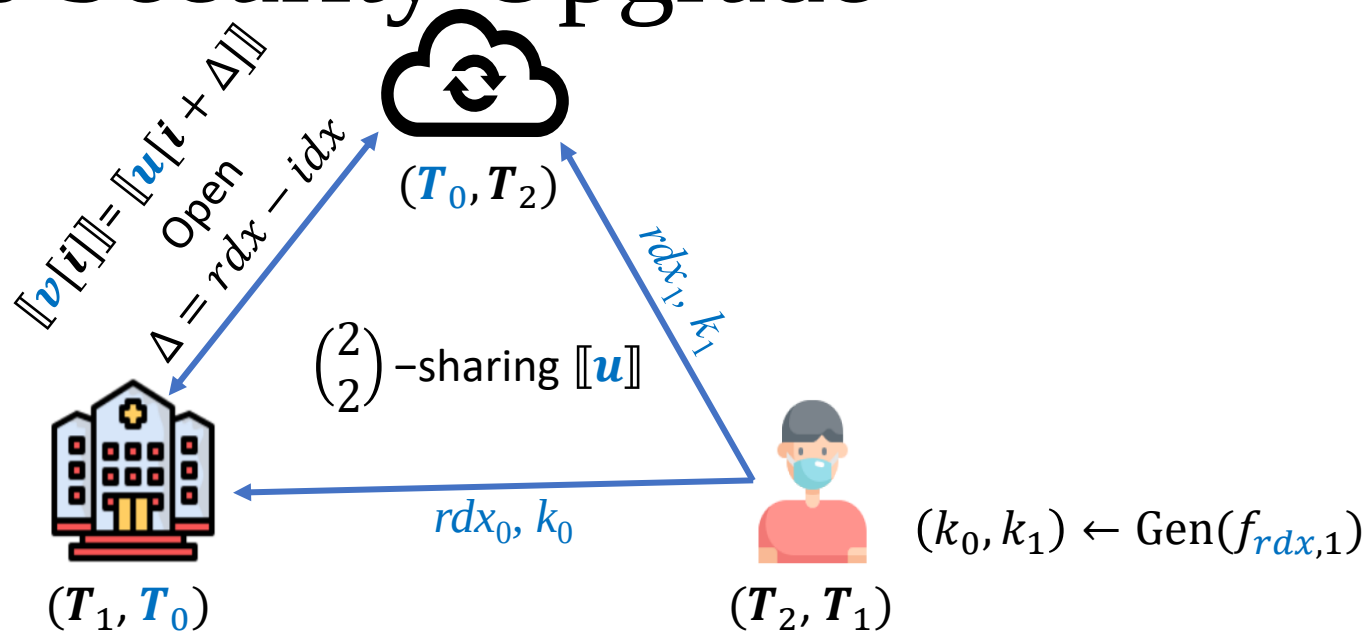
- Pair-wise two-party inner-product computation
  - Input: $\langle \boldsymbol{T} \rangle$ and $\langle idx \rangle$
  - Output: $\binom{3}{3}$–sharing $[\![\boldsymbol{T}[idx]]\!]$

Malicious Security Upgrade



- How to check DPF keys are correctly correlated?
  - Solution: Use Verifiable DPF (VDPF)
- How to check $[\![rdx]\!]$ is consistent with DPF keys?
  - Two evaluating parties check the following equations:
    - $rdx - \sum_i i \cdot \boldsymbol{u}[i] = 0$
- How to open Δ correctly?
  - Non-interactive conversion from $[\![rdx]\!]$ to $\langle rdx \rangle$
  - $\langle \Delta \rangle \leftarrow \langle rdx \rangle - \langle idx \rangle$

Malicious Security Upgrade



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- How to open Δ correctly?
  - Non-interactive conversion from $[\![rdx]\!]$ to $\langle rdx \rangle$
  - $\langle \Delta \rangle \leftarrow \langle rdx \rangle - \langle idx \rangle$

$P_0: (0, rdx_1)$
$P_1: (rdx_0, 0)$
$P_2: (rdx_1, rdx_0)$

Detect Additive Attacks

- Oblivious selection up to additive errors

$$\langle \boldsymbol{T}[idx] + \boldsymbol{e} \rangle \leftarrow F_{\mathrm{OS}}(\langle \boldsymbol{T} \rangle, \langle idx \rangle)$$

- $\mathrm{DTE}(T, X)$:
  - $idx \leftarrow 0, result \leftarrow \perp$
  - For $i \in [1, d']$: $//d' \geq d$
    - $(t, l, r, v, c) \leftarrow T[idx]$
    - $x \leftarrow X[v]$
    - $b \leftarrow x < t$
    - $idx \leftarrow r \oplus b \cdot (r \oplus l)$
    - $result \leftarrow c$
  - Return $result$

Detect Additive Attacks

- Oblivious selection up to additive errors

$$\langle \boldsymbol{T}[idx] + \boldsymbol{e} \rangle \leftarrow F_{\mathrm{OS}}(\langle \boldsymbol{T} \rangle, \langle idx \rangle)$$

- Solution: SPDZ-like MAC
  - Given a shared value x, a SPDZ-like MAC is defined as $m(x) = \alpha \cdot x$ over a finite filed **F**
  - $m(x)$ is shared along with x

- DTE(T, X):
  - $idx \leftarrow 0, result \leftarrow \perp$
  - For $i \in [1, d']$: $//d' \geq d$
    - $(t, l, r, v, c) \leftarrow T[idx]$
    - $x \leftarrow X[v]$
    - $b \leftarrow x < t$
    - $idx \leftarrow r \oplus b \cdot (r \oplus l)$
    - $result \leftarrow c$
  - Return $result$

Detect Additive Attacks

- Oblivious selection up to additive errors

$$\langle \boldsymbol{T}[idx] + \boldsymbol{e} \rangle \leftarrow F_{\mathrm{OS}}(\langle \boldsymbol{T} \rangle, \langle idx \rangle)$$

- Solution: SPDZ-like MAC
  - Given a shared value x, a SPDZ-like MAC is defined as $m(x) = \alpha \cdot x$ over a finite filed **F**
  - $m(x)$ is shared along with x
- Our MAC scheme:
  - $m(x) = \alpha \cdot x$ over $\mathbf{F}_{2^l}$; l is the bit-length of elements in $\boldsymbol{T}$.

$$\langle \boldsymbol{T}[idx] + \boldsymbol{e} \rangle \leftarrow F_{\mathrm{OS}}(\langle \boldsymbol{T} \rangle, \langle idx \rangle)$$
$$\langle m(\boldsymbol{T}[idx]) + \boldsymbol{e'} \rangle \leftarrow F_{\mathrm{OS}}(\langle m(\boldsymbol{T}) \rangle, \langle idx \rangle)$$

- DTE(T, X):
  - $idx \leftarrow 0, result \leftarrow \perp$
  - For $i \in [1, M]$: $//M \geq d$
    - $(t, l, r, v, c) \leftarrow T[idx]$
    - $x \leftarrow X[v]$
    - $b \leftarrow x < t$
    - $idx \leftarrow r \oplus b \cdot (r \oplus l)$
    - $result \leftarrow c$
  - Return $result$

Malicious PDTE with Sublinear Communication

- Sublinear (online) communication
  - Tree encoding with optimizations.
  - Two OS protocols with constant online communication.

- Malicious security
  - Three-party honest-majority setting.
  - Lightweight consistency checks.
  - *Privacy*: Only the FO learns the classification result.
  - *Correctness*: Mostree always outputs the correct classification.

Experiment Setting

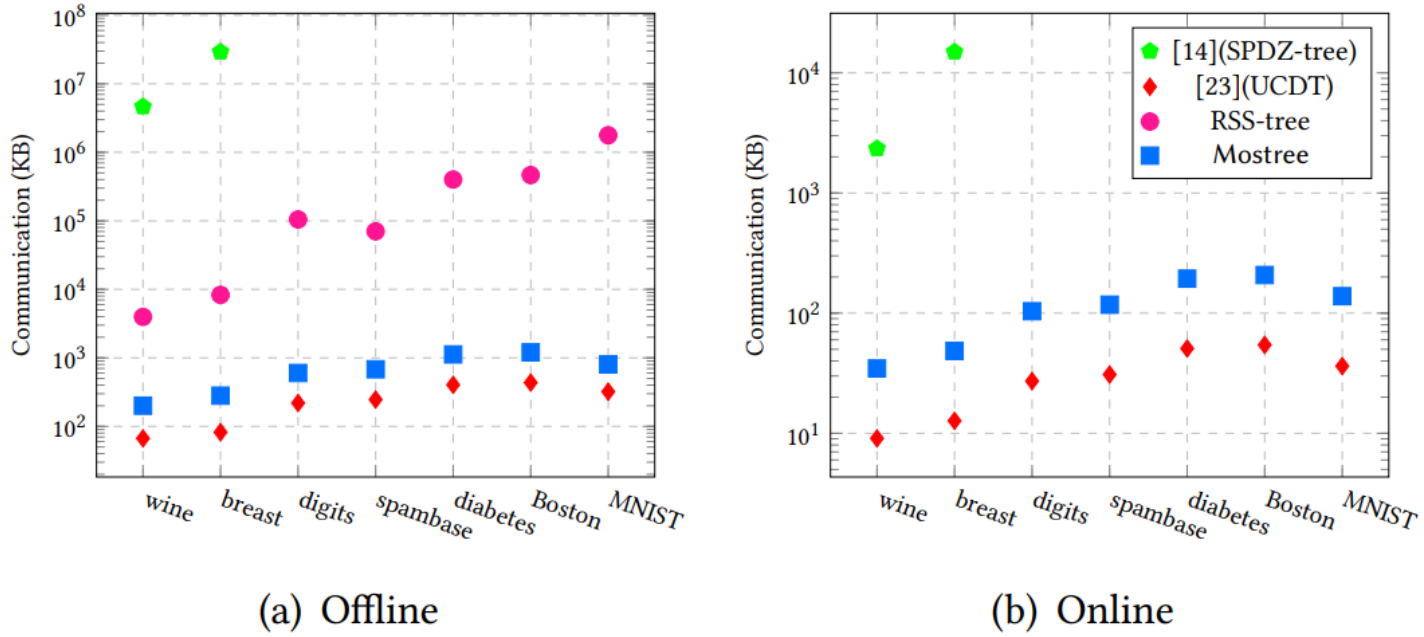
- Machine: Intel(R) Core i9-9900 CPU, 3.10 GHz×16, Ubuntu 20.04 LTS with 32 GB of memory
- Network setting:
  - LAN (RTT: 0.1ms, bandwidth: 1 Gbps)
  - MAN (RTT: 6ms, bandwidth: 100 Mbps)
  - WAN (RTT: 80ms, bandwidth: 40 Mbps)
- Dataset: UCI repository

Tree	Feature Dimension	Depth	# of Nodes
Wine	7	5	23
Linnerud	3	6	39
Breast	12	7	43
Digits	47	15	337
Spambase	57	17	171
Diabetes	10	28	787
Boston	13	30	851
MNIST	784	20	4179

https://github.com/Jbai795/Mostree-pub.git

Performance

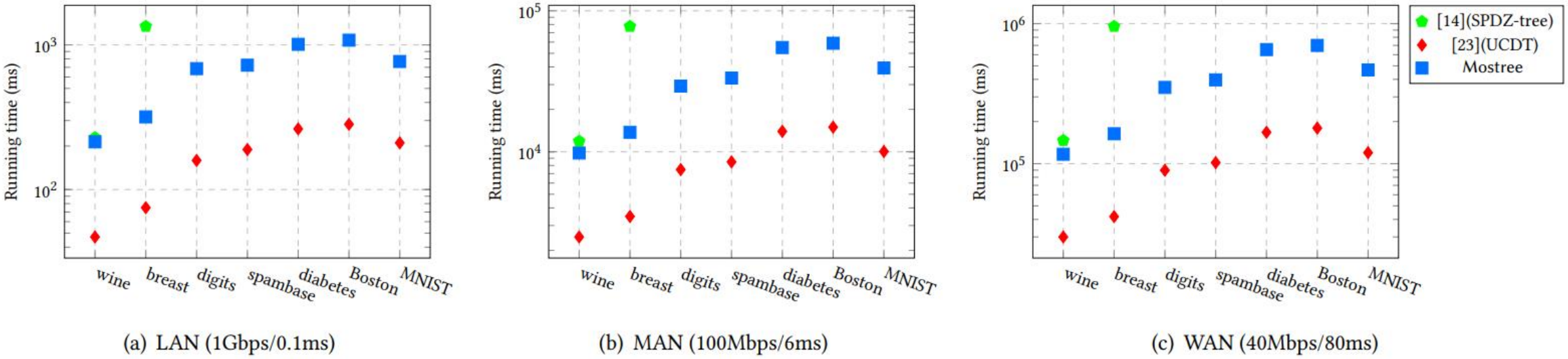
- Online communication



- Mostree requires ~4x online communication than UCDT over MNIST.
- RSS-Tree requires much more offline communication than Mostree.

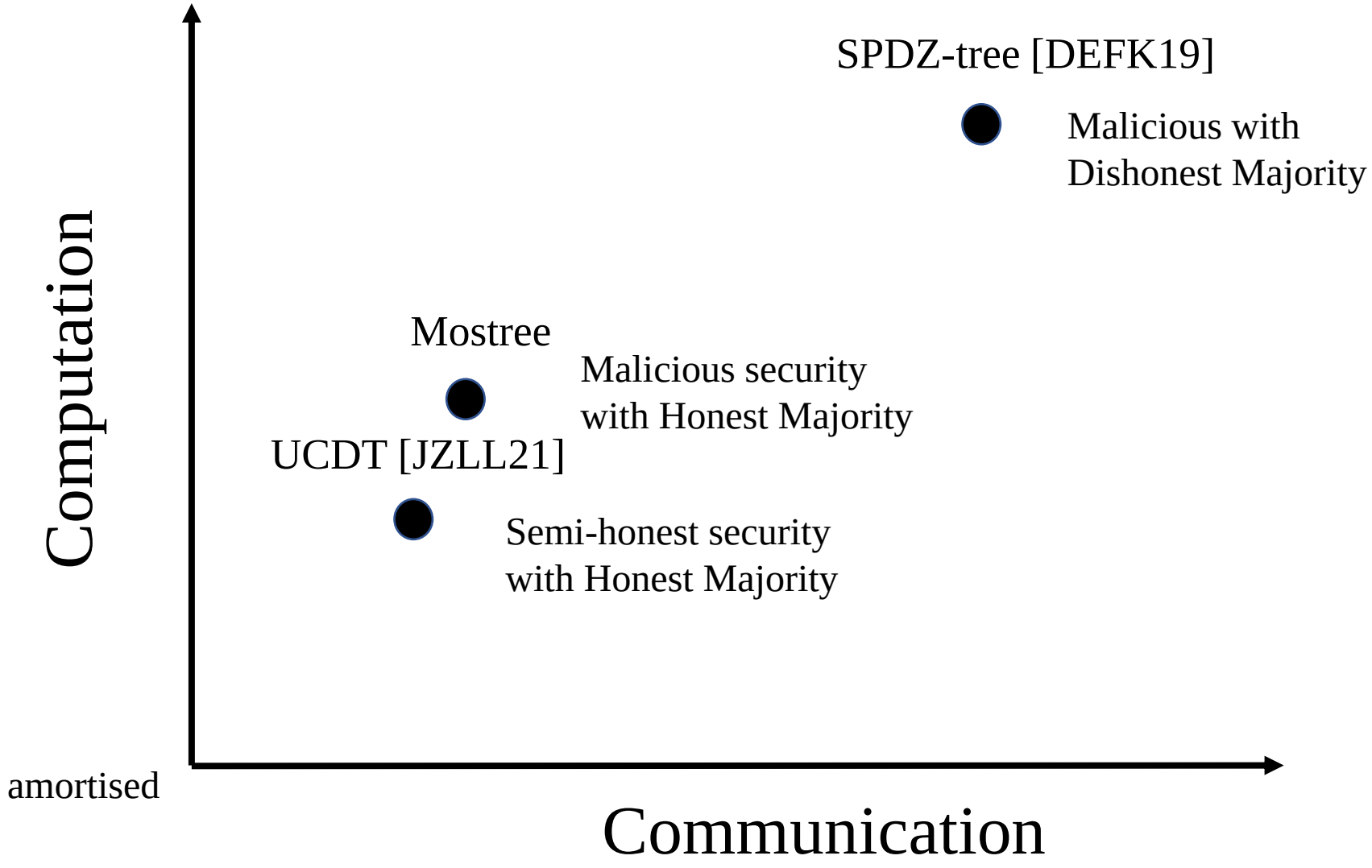
Performance

- Online running time



- ~4x overhead than the semi-honest version.
- In the LAN setting, Mostree requires ~0.7s running time over MNIST. ☺
- In the WAN setting, Mostree requires ~467s over MNIST. ☹

Comparison of Approaches



Conclusion

- We present a cryptographic solution to private decision tree evaluation problem.
- At the core of our scheme are oblivious selection protocols with sublinear (online) communication.
- Lightweight consistency check to ensure malicious security.

- Future work
  - Improving efficiency in the WAN setting → reducing round complexity.
  - Sublinear computation?
  - Applications to other areas, e.g., private database applications.

Thanks for your attention!