Annual Computer Security Applications Conference 2023

Secure Softmax/Sigmoid for Machine-learning Computation

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Rundown

- <u>Secure</u> Machine Learning Background
  - Secret share: 2 computing parties  + 1 commodity server 
  - Against semi-honest adversary

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- Non-Linearity Challenges and Sigmoid/Softmax in Crypto
- **New Protocols** for Nonlinear Functions
  - Local-sigmoid via Fourier series
  - Quasi-softmax via ordinary differential equation

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- **New Protocols** for Nonlinear Functions
  - Local-sigmoid via Fourier series
  - Quasi-softmax via ordinary differential equation
- Experiments and System Performance
- Conclusion

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- Training is more complicated to do with cryptography
  - It produces fluctuating computation results.
  - It requires non-linear computation such as those in activation layers.

Crypto. Challenges in Secure Training

- Crypto. excels primarily with *finite fields* and *linear functions*.
  - **Accuracy**: *expand* finite field to cater to fluctuating ranges.
- But, increase computational & **communication** overheads.
  - Secure protocols for exact computation of non-linearity are known to be heavyweight.

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- But, increase computational & **communication** overheads.
  - Secure protocols for exact computation of non-linearity are known to be heavyweight.
- Not until recently, start to have secure training frameworks.
  - CrypTen [NeurIPS'21], CryptGPU [S&P’21], Piranha [Usenix Sec’22], etc.
  - Support more complex activation, including softmax and sigmoid.
  - Achieve high computational performance over AlexNet (60M param) and VGG-16 (138M param) .

Communication Bottleneck

- However, large communication overhead persists as a major concern.
  - Prominently, Piranha, a GPU platform for secure computation, reports **94%+** of the training time consumed by communication in a wide-area network (WAN) setting.

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- Informally, sigmoid/softmax combine e^x , $1/x$, or Σe^x.
  - e^x and $1/x$ are unbounded and continuous.
- Securely computing them with efficiency is challenging.
  - Existing works either separately approximate or replace them.
  - Below, we detail secure sigmoid and softmax one by one.

State-of-The-Art Secure Sigmoid

- **Sigmoid(**x**) =** $e^x/(e^x + 1)$: (-inf, +inf) ↦ **[0, 1].**
  - Binary classification.
  - It squashes any input to a value in (0,1).

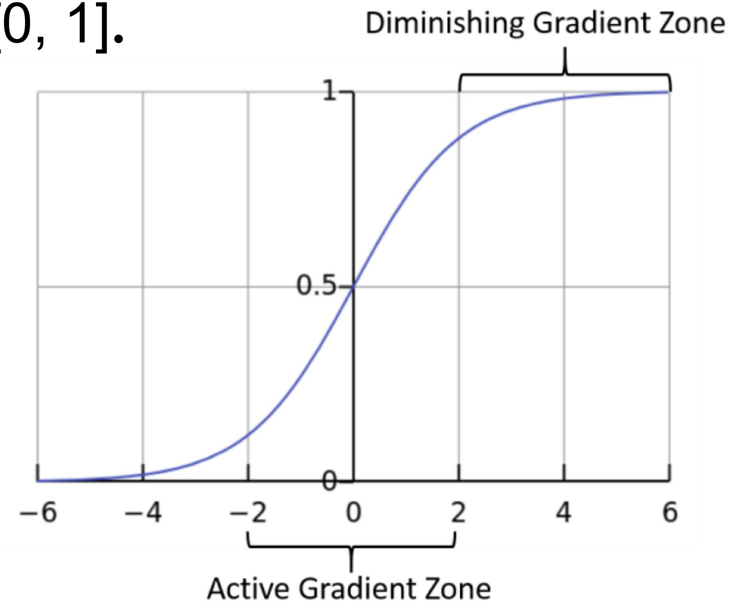

*Figure is from Google image.

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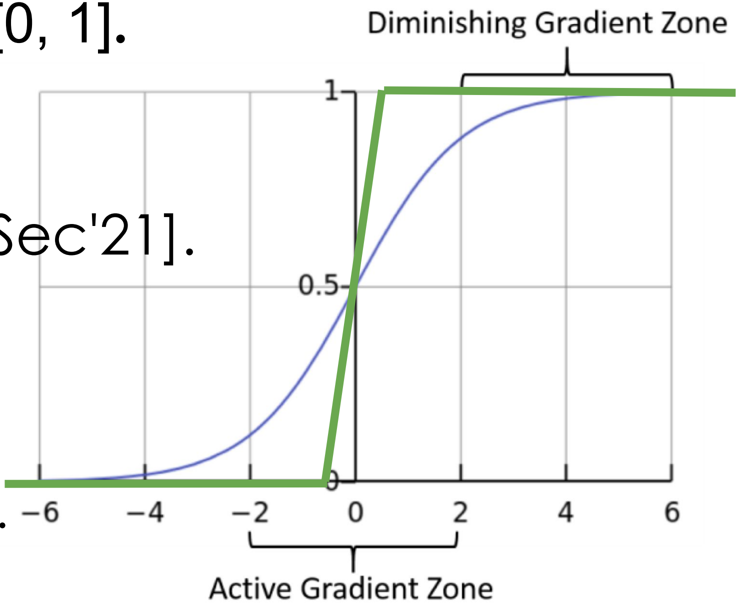
- ABY-series protocols [CCS'18, Usenix Sec'21].
  - Piecewise linear approx.
  - $x + 0.5$ for $|x| < 0.5$; $x = 0$ or $x = 1$.
  - Requires comparison to identify pieces.
  - Relatively inaccurate approx. near to 0.



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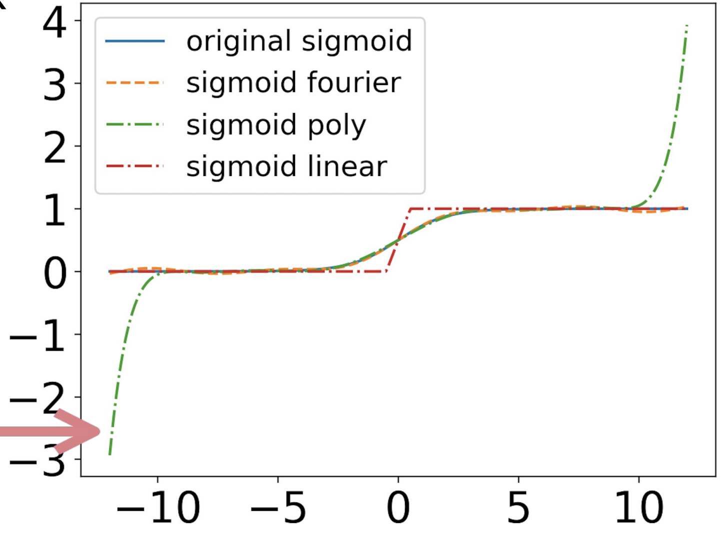
- Chebyshev polynomial
  - [CCS'21 Workshop]
  - Linear to #iterms
  - Possibly result in gradient explosion



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- How about their communication costs?

Protocol	Offline (bits)	Online (bits)	Overall (bits)	Round
ABY-series	-	~800	-	5
Polyn. (5,8)	320 ~ 512	1280 ~ 2048	1600 ~ 2560	1

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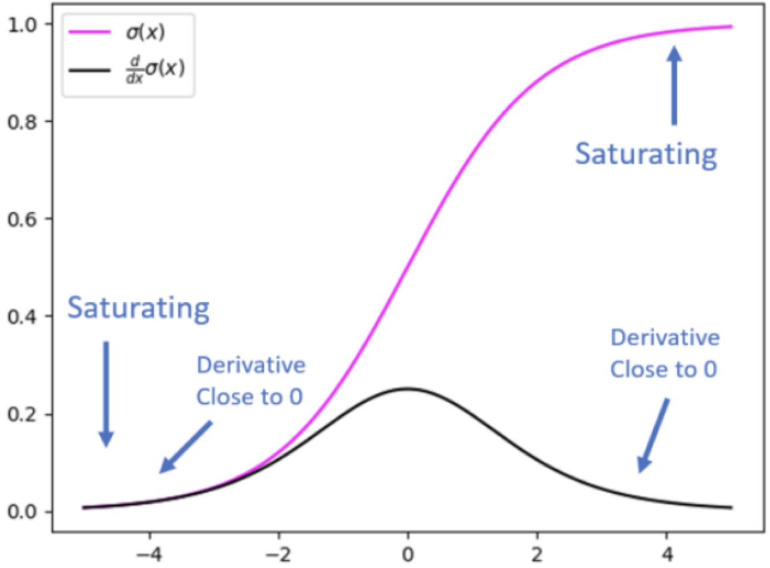
New Sigmoid Approx. and Protocol

- Local-Sigmoid definition.
  - Sigmoid in [-*a*, *a*].
  - High accuracy in range.
  - Bounded error out of range

Sigmoid Function and Its Derivative



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New Sigmoid Approx. and Protocol

- Local-Sigmoid definition.
  - Sigmoid in [-a, a].
  - High accuracy in range.
  - Bounded error out of range
- Fourier approximation.
  - LSig(x) = a + **b**sin(x***k***).
  - Mask t, shared value $\Delta = x - t$.
  - No secure comparison is required.
  - $\sin(\Delta + t) = \sin(\Delta)\cos(t) + \cos(\Delta)\sin(t)$

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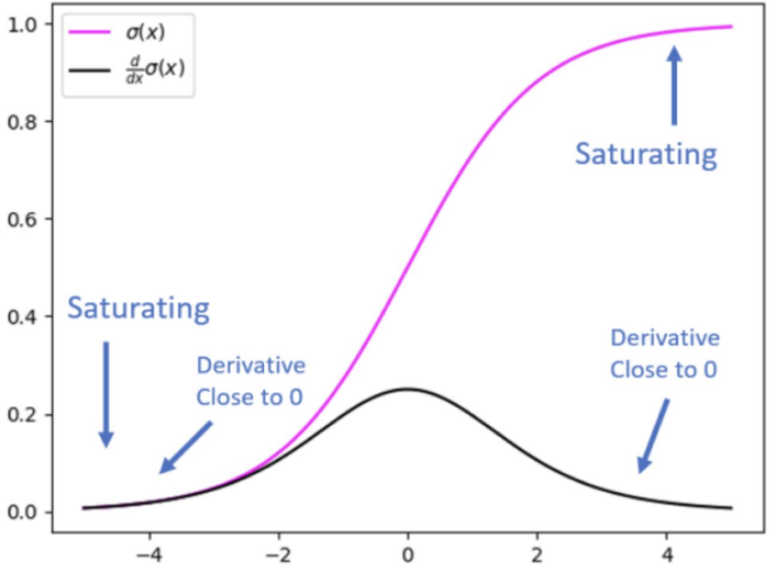
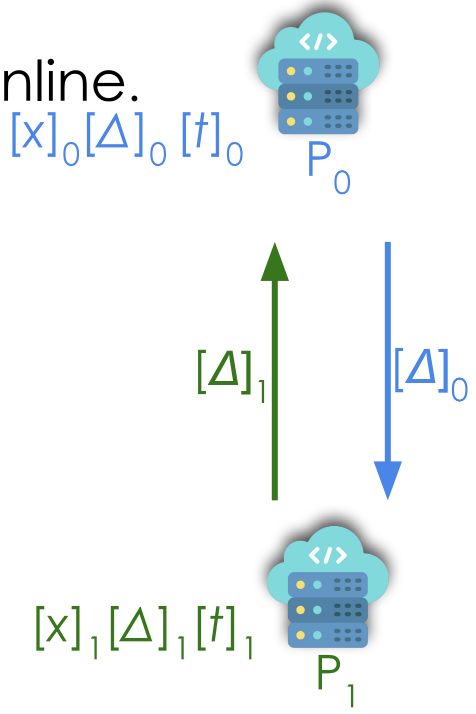


New Sigmoid Approx. and Protocol

- Jointly compute LSig(x) = $a + \boldsymbol{b}\sin((\Delta + t)\boldsymbol{k})$ online.
  - Public parameters a, $\boldsymbol{b}$, $\boldsymbol{k}$.
  - P_0 holds $[\Delta]_0 = [x]_0 - [t]_0$; P_1 holds $[\Delta]_1 = [x]_1 - [t]_1$.
  - P_0 sends $[\Delta]_0$; P_1 sends $[\Delta]_1$. [1 round]



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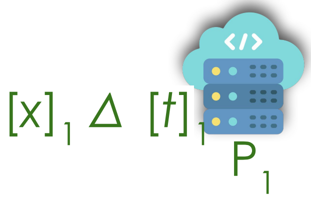

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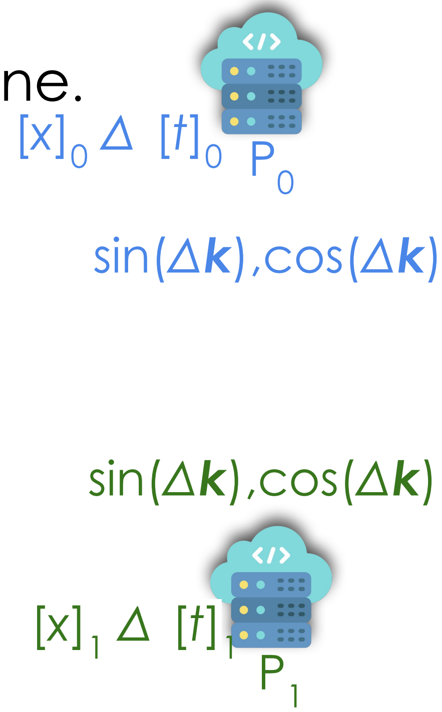

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- Secret-shared outputs
  - P_0 gets $a + \boldsymbol{b}(\sin(\Delta\boldsymbol{k})[v]_0 + \cos(\Delta\boldsymbol{k})[u]_0)$.
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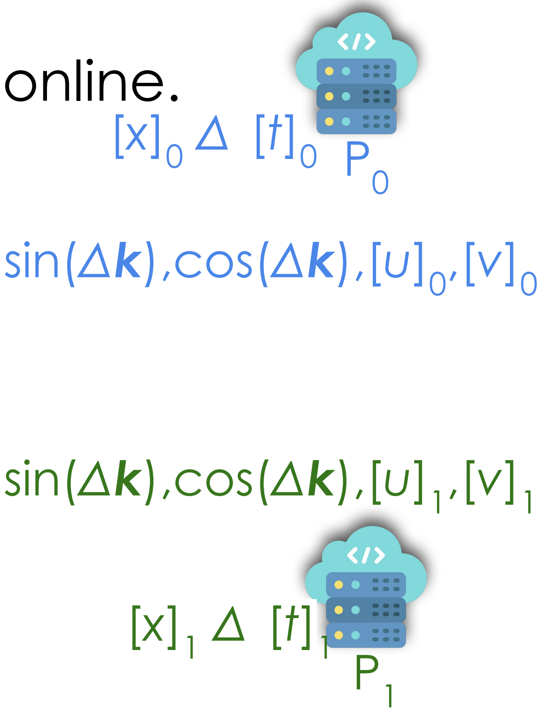

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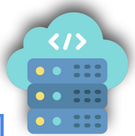
  - P_1 gets $a + \boldsymbol{b}(\sin(\Delta\boldsymbol{k})[v]_1 + \cos(\Delta\boldsymbol{k})[u]_1)$.
- What are $[u]_0, [v]_0, [t]_0$ and $[u]_1, [v]_1, [t]_1$?



$[x]_0$ Δ $[t]_0$ P_0

$\sin(\Delta\boldsymbol{k}), \cos(\Delta\boldsymbol{k}), [u]_0, [v]_0$

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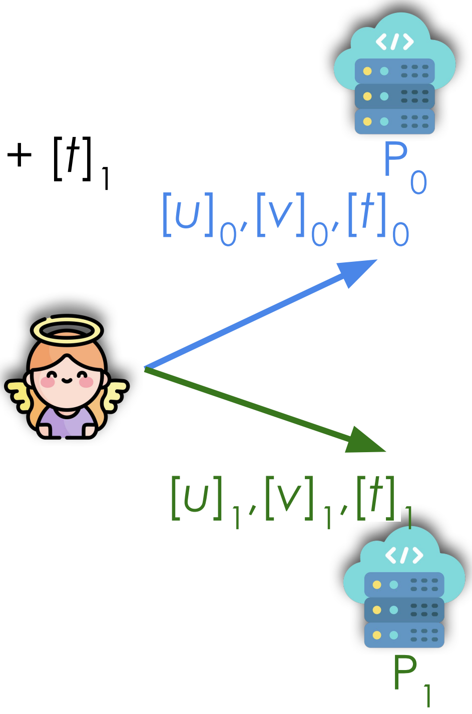
- What are $[u]_0, [v]_0, [t]_0$ and $[u]_1, [v]_1, [t]_1$?
  - $\sin(t\mathbf{k}) = [u]_0 + [u]_1$; $\cos(t\mathbf{k}) = [v]_0 + [v]_1$; $t = [t]_0 + [t]_1$
  - Randomness independent to private x.
  - Generated by a crypto commodity server.
  - In a pre-computation phase offline.



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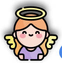
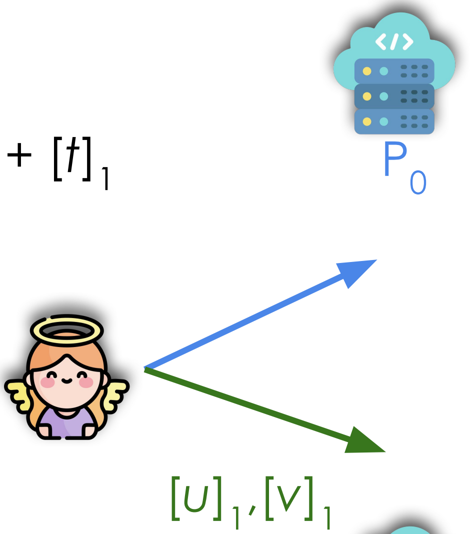
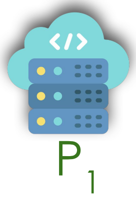
- Optimize offline communication?
  - Use PRF with a synchronized counter.
  - &P_0 generate $[u]_0, [v]_0, [t]_0$ using the same key_0.
  - &P_1 generate $[t]_1$ using the same key_1.
  -  computes and sends $[u]_1, [v]_1$ to P_1.





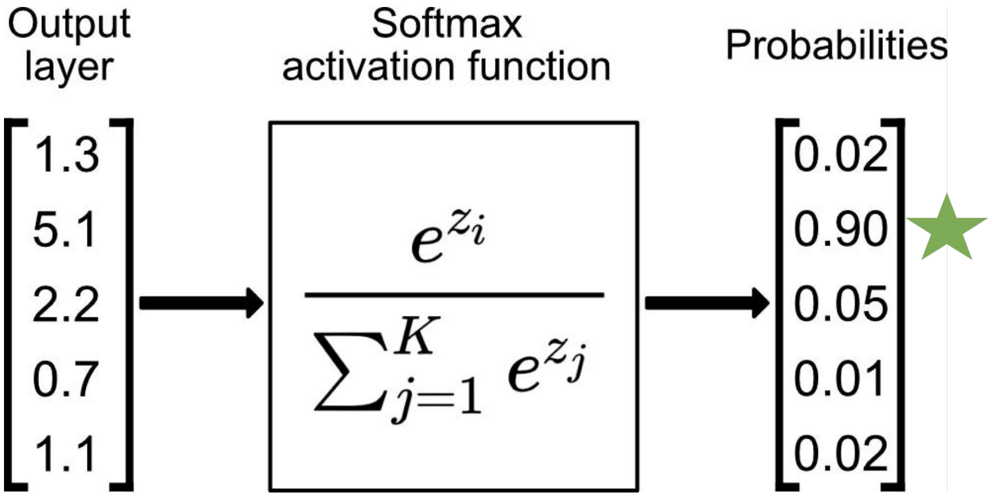
Communication Costs

- Achieved **< 40bits** online in **1** round!

Protocol	Offline (bits)	Online (bits)	Overall (bits)	Round
ABY-series	-	∼800	-	5
Polyn. (K=5,8)	320 ∼ 512	1280 ∼ 2048	1600 ∼ 2560	1
Ours (m=4, K=5)	640	**36**	676	**1**
Ours (m=4, K=8)	1024	**36**	1060	**1**
Ours (m=5, K=5)	640	**38**	678	**1**
Ours (m=5, K=8)	1024	**38**	1062	**1**

State-of-The-Art Secure Softmax

- **Softmax(**x**)** = $e^{z\_i}/(\Sigma e^{z\_j})$: $R^m \mapsto [0, 1]^m$.
  - It squashes any input vector to a probability vector.
  - Multi-classification.



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  - Secure maximum takes O(log m) rounds for removing the largest input and mitigating overflow of e^{z_i} and Σe^{z_j}.

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  - It requires secure maximum, exponentiation, and division.
  - Secure maximum takes O(log m) rounds for removing the largest input and mitigating overflow of e^{z_i} and Σe^{z_j}.
- ASM protocol replaces exponential function with ReLU.
  - It is adopted in SecureNN [PoPETS'19], Falcon [PoPETS'21].
  - It relies on manual efforts in tuning the model [Keller and Sun, ICML'22].

State-of-The-Art Secure Softmax

- How about their communication costs?

Protocol	#Class	Online (bits)	Overall (bits)	Round
ASM protocol	10	-	3M	704
ASM protocol	100	-	30M	704
ASM protocol	1000	-	302M	704
Crypten	10	783250	982K	171
Crypten	100	8536390	11M	300
Crypten	1000	86067790	108M	430

State-of-The-Art Secure Softmax

- How a

Can we expect **<10% communication costs** in **<35** rounds?

Protoco	[illegible]	[illegible]	[illegible]	nd
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- For an input vector **x** and iteration step *r*, we instantiate a <u>vector</u> function QSMax ***g****()*, which is an ordinary differential equation solved by Euler formula.
  - $\mathbf{g}(0) = [1/m, \ldots, 1/m]$

    for $i = 1, \ldots, r$ **do**

    $\mathbf{g}(i/r) = \mathbf{g}((i-1)/r) + (\mathbf{x} - \langle \mathbf{x}, \mathbf{g}((i-1)/r) \rangle \mathbf{1}) * \mathbf{g}((i-1)/r)/r$
- $\mathbf{g}(r/r) = \mathbf{g}(1)$, *iteratively limits to real softmax.*

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    g(*i*/*r*) = ***g***((*i*-1)/*r*) + (**x** - <**x**,***g***((*i*-1)/*r*)>**1**)****g***((*i*-1)/*r*)/*r*
- ***g***(*r*/*r*) = ***g***(1), *iteratively limits to real softmax.*
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$[1/m, \ldots, 1/m]$

P_0

$[0, \ldots, 0]$

P_1

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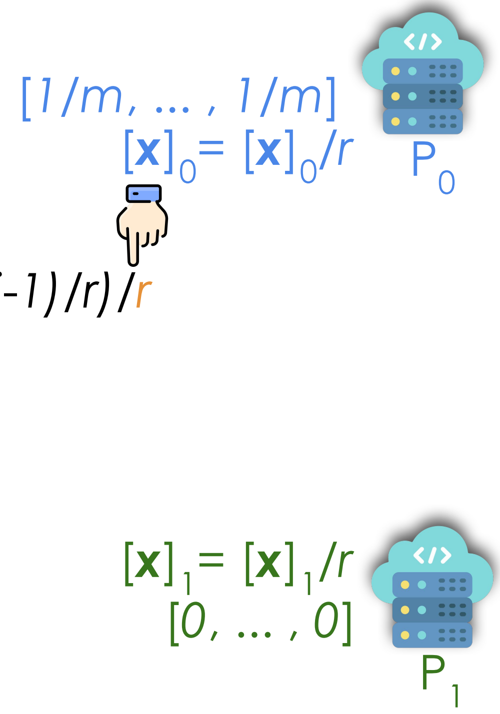

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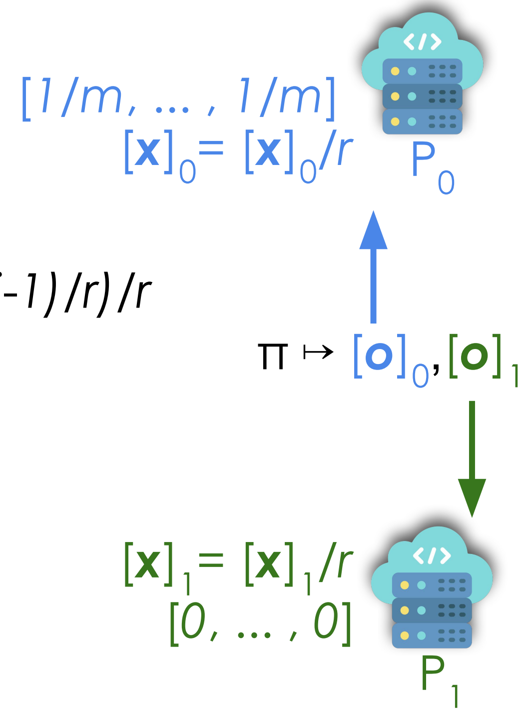
- Jointly invoke secure multiplication π.
  - Inner product with m dimensions parallelly.



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$[1/m, \dots, 1/m]$
$[\mathbf{x}]_0 = [\mathbf{x}]_0 / r$
$[\mathbf{q}]_0 = (\Sigma[\mathbf{p}_i]_0)\,\mathbf{1}$

P_0

$[\mathbf{q}]_1 = (\Sigma[\mathbf{p}_i]_1)\,\mathbf{1}$
$[\mathbf{x}]_1 = [\mathbf{x}]_1 / r$
$[0, \dots, 0]$

P_1

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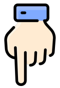

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$[1/m, \ldots, 1/m]$

$[\mathbf{x}]_0 = [\mathbf{x}]_0/r$

$[\mathbf{w}]_0 = [\mathbf{x}]_0 - [\mathbf{q}]_0$

P_0

$[\mathbf{w}]_1 = [\mathbf{x}]_1 - [\mathbf{q}]_1$

$[\mathbf{x}]_1 = [\mathbf{x}]_1/r$

$[0, \ldots, 0]$

P_1

New Softmax Approx. and Protocol

- Jointly compute QSMax(x) online
  - $\mathbf{g}(0) = [1/m, \dots, 1/m]$

    for $i = 1, \dots, r$ **do**

    $$\mathbf{g}(i/r) = \mathbf{g}((i-1)/r) + (\mathbf{x} - \langle \mathbf{x}, \mathbf{g}((i-1)/r)\rangle \mathbf{1}) * \mathbf{g}((i-1)/r)/r$$
- Jointly invoke secure multiplication π.
  - Inner product with *m* dimensions parallelly.
  - Element-wise product with *m* dimensions.

$[1/m, \dots, 1/m]$

$[\mathbf{x}]_0 = [\mathbf{x}]_0/r$

$[\mathbf{w}]_0 = [\mathbf{x}]_0 - [\mathbf{q}]_0$

P_0

$\pi \mapsto [\mathbf{t}]_0, [\mathbf{t}]_1$

$[\mathbf{w}]_1 = [\mathbf{x}]_1 - [\mathbf{q}]_1$

$[\mathbf{x}]_1 = [\mathbf{x}]_1/r$

$[0, \dots, 0]$

P_1

New Softmax Approx. and Protocol

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- Jointly invoke secure multiplication π.
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  - Element-wise product with m dimensions.

$[1/m, \ldots, 1/m]$

$[\mathbf{x}]_0 = [\mathbf{x}]_0 / r$

P_0

$[\mathbf{p}]_0 = [\mathbf{x}]_0 - [\mathbf{q}]_0$

$[\mathbf{g}((i-1)/r)]_0 + [\mathbf{t}]_0$

$[\mathbf{g}((i-1)/r)]_1 + [\mathbf{t}]_1$

$[\mathbf{p}]_1 = [\mathbf{x}]_1 - [\mathbf{q}]_1$

$[\mathbf{x}]_1 = [\mathbf{x}]_1 / r$

$[0, \ldots, 0]$

P_1

New Softmax Approx. and Protocol

- Jointly compute QSMax(x) online
  - $\mathbf{g}(0) = [1/m, \ldots, 1/m]$
    for $i = 1, \ldots, r$ **do**
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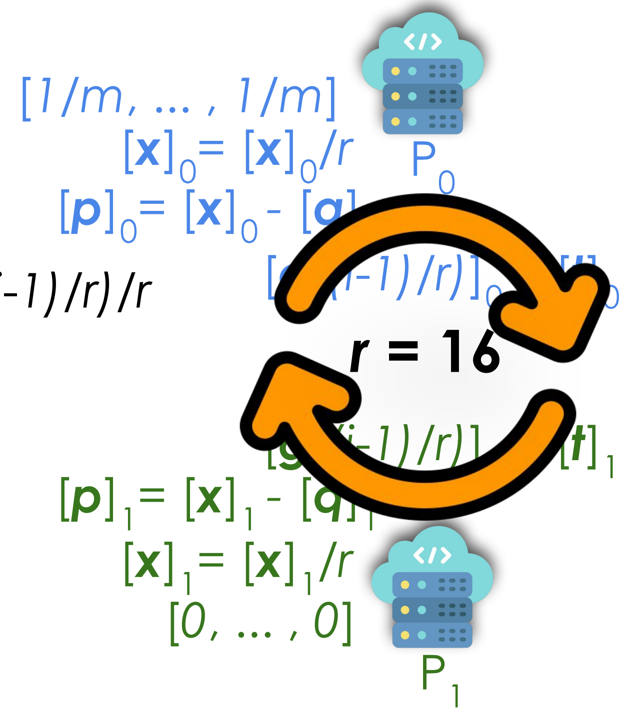
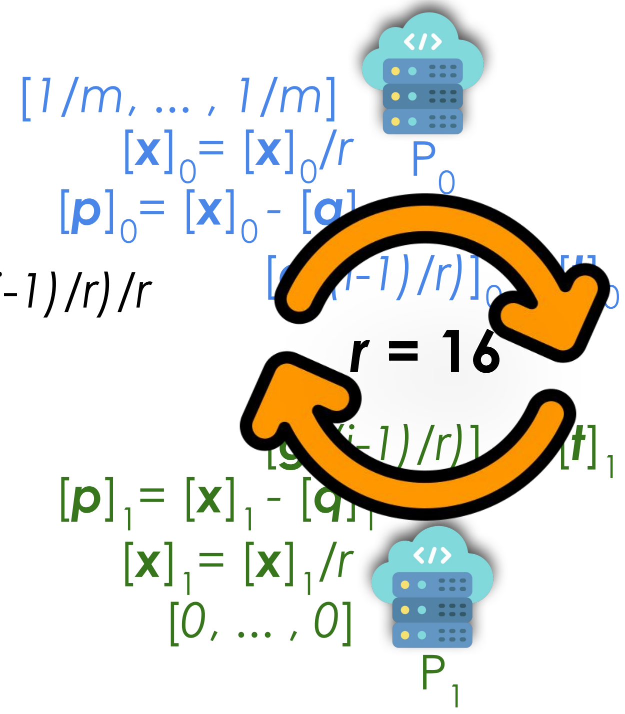
- Jointly invoke secure multiplication π.
  - Inner product with m dimensions parallelly.
  - Element-wise product with m dimensions.



New Softmax Approx. and Protocol

- Jointly compute QSMax(x) online
  - $\mathbf{g}(0) = [1/m, \dots, 1/m]$
    for $i = 1, \dots, r$ **do**
      $\mathbf{g}(i/r) = \mathbf{g}((i-1)/r) + (\mathbf{x} - \langle \mathbf{x}, \mathbf{g}((i-1)/r) \rangle \mathbf{1}) * \mathbf{g}((i-1)/r)/r$
- Jointly invoke secure multiplication π.
  - Inner product with *m* dimensions parallelly.
  - Element-wise product with *m* dimensions.
- Secret-shared outputs.
  - P_0 gets $[\mathbf{g}(1)]_0$.
  - P_1 gets $[\mathbf{g}(1)]_0$.



Communication Costs

- Achieved **< 10%** communication costs in **32** rounds!

Protocol	#Class	Online (bits)	Overall (bits)	Round
ASM protocol	10	-	3M	704
Crypten	10	783K	982K	171
Ours	10	63K	**84K**	**32**
ASM protocol	100	-	30M	704
Crypten	100	8.5M	11M	300
Ours	100	616K	**821K**	**32**
ASM protocol	1000	-	302M	704
Crypten	1000	86M	108M	430
Ours	1000	6M	**8M**	**32**

Experiments & System Performance

- Datasets: MNIST, CIFAR-10
- Models: AlexNet, LeNet, VGG-16, ResNet, Networks A-B-C-D.
- Communication reduces by **57%-77%.**
- Accuracy
  - reaches a higher accuracy for AlexNet, VGG-16 compared with Piranha [Usenix Sec'22].
  - reaches a similar accuracy for Networks A-B-C-D compared with SPDZ-QT [Keller and Sun, ICML'22].
- Training time
  - **10%-60%** speed-up in LAN & **56%-78%** speed-up in WAN.

Conclusion

- Propose two cryptography-friendly approximations for secure computation of softmax and sigmoid, leading to expedited private training with much lower communication.
- Provide both C++ & Python implementation for different programming preference.
- Shed light on protocol design for bounded nonlinear functions, avoiding unbounded intermediate functions (e^x, $1/x$).
- Extend the realm of secure computation to encompass solutions for differential equations with rational polynomial or trigonometric functions coefficients.